Q1.

Nodes and branches are the basic elements of signal flow graph.

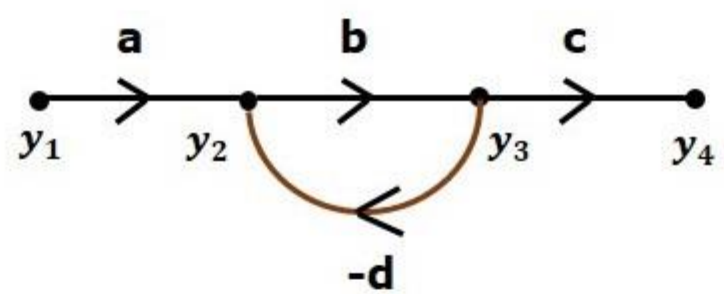
Node

Node is a point which represents either a variable or a signal. There are three types of nodes — input node, output node and mixed node.

- **Input Node** − It is a node, which has only outgoing branches.
- **Output Node** − It is a node, which has only incoming branches.
- **Mixed Node** − It is a node, which has both incoming and outgoing branches.

Example

Let us consider the following signal flow graph to identify these nodes.



- The **nodes** present in this signal flow graph are $\mathbf{y_1}$**,** $\mathbf{y_2}$**,** $\mathbf{y_3}$ and $\mathbf{y_4}$**.**
- $\mathbf{y_1}$ and $\mathbf{y_4}$ are the **input node** and **output node** respectively.
- $\mathbf{y_2}$ and $\mathbf{y_3}$ are **mixed nodes**.

Branch

Branch is a line segment which joins two nodes. It has both **gain** and **direction**. For example, there are four branches in the above signal flow graph. These branches have **gains** of **a, b, c** and **-d**.

Construction of Signal Flow Graph

Let us construct a signal flow graph by considering the following algebraic equations −

$$y_2=a_{12}y_1+a_{42}y_4 \quad y2=a12y1+a42y4$$

$$y_3=a_{23}y_2+a_{53}y_5 \quad y3=a23y2+a53y5$$

$$y_4=a_{34}y_3 \quad y4=a34y3$$

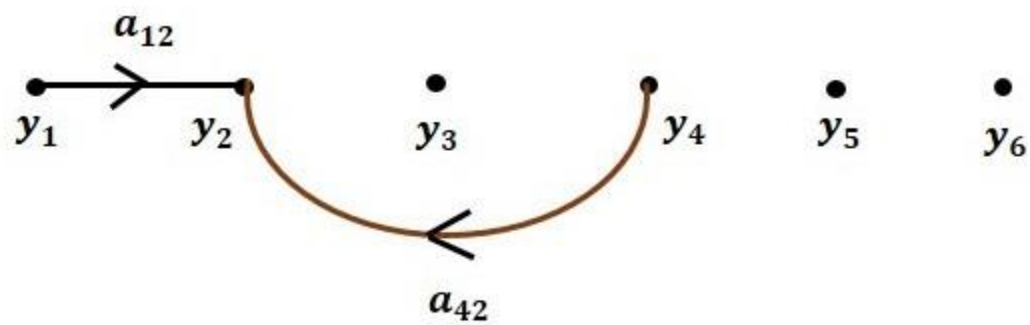
$y_5=a_{45}y_4+a_{35}y_3$y5=a45y4+a35y3

$y_6=a_{56}y_5$y6=a56y5

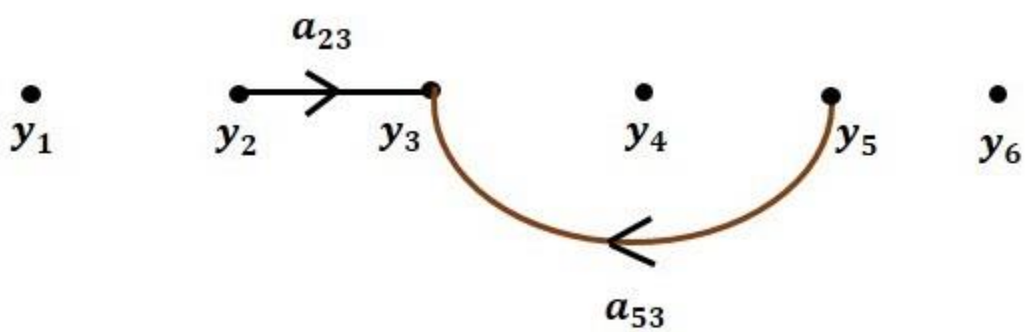
There will be six **nodes** (y_1, y_2, y_3, y_4, y_5 and y_6) and eight **branches** in this signal flow graph. The gains of the branches are a_{12}, a_{23}, a_{34}, a_{45}, a_{56}, a_{42}, a_{53} and a_{35}.

To get the overall signal flow graph, draw the signal flow graph for each equation, then combine all these signal flow graphs and then follow the steps given below −

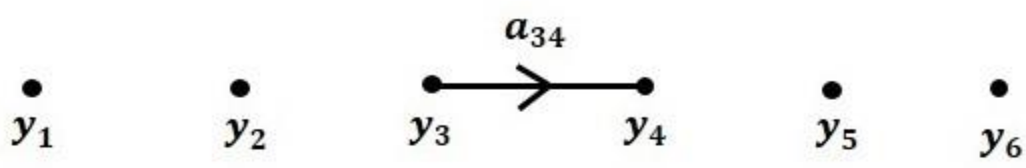
Step 1 − Signal flow graph for $y_2=a_{13}y_1+a_{42}y_4$y2=a13y1+a42y4 is shown in the following figure.



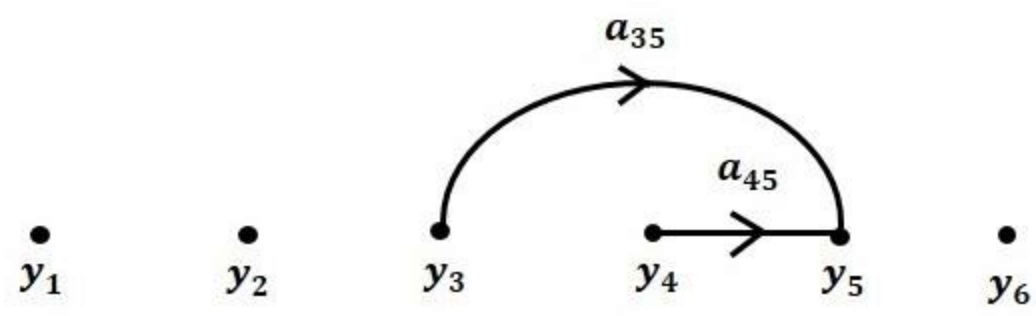
Step 2 − Signal flow graph for $y_3=a_{23}y_2+a_{53}y_5$y3=a23y2+a53y5 is shown in the following figure.



Step 3 − Signal flow graph for $y_4=a_{34}y_3$y4=a34y3 is shown in the following figure.

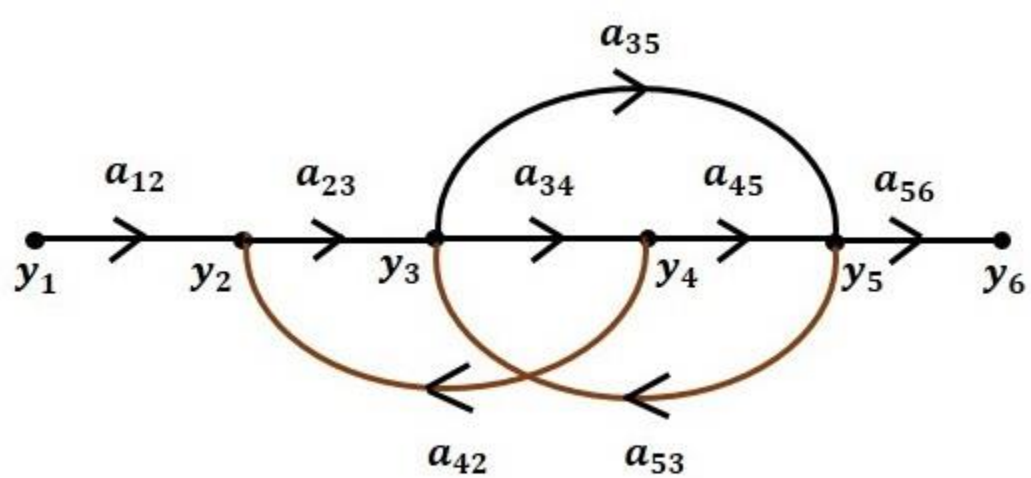


Step 4 − Signal flow graph for $y_5=a_{45}y_4+a_{35}y_3$y5=a45y4+a35y3 is shown in the following figure.



Step 5 − Signal flow graph for $y_6=a_{56}y_5y_6=a_{56}y_5$ is shown in the following figure.

Step 6 − Signal flow graph of overall system is shown in the following figure.



Conversion of Block Diagrams into Signal Flow Graphs

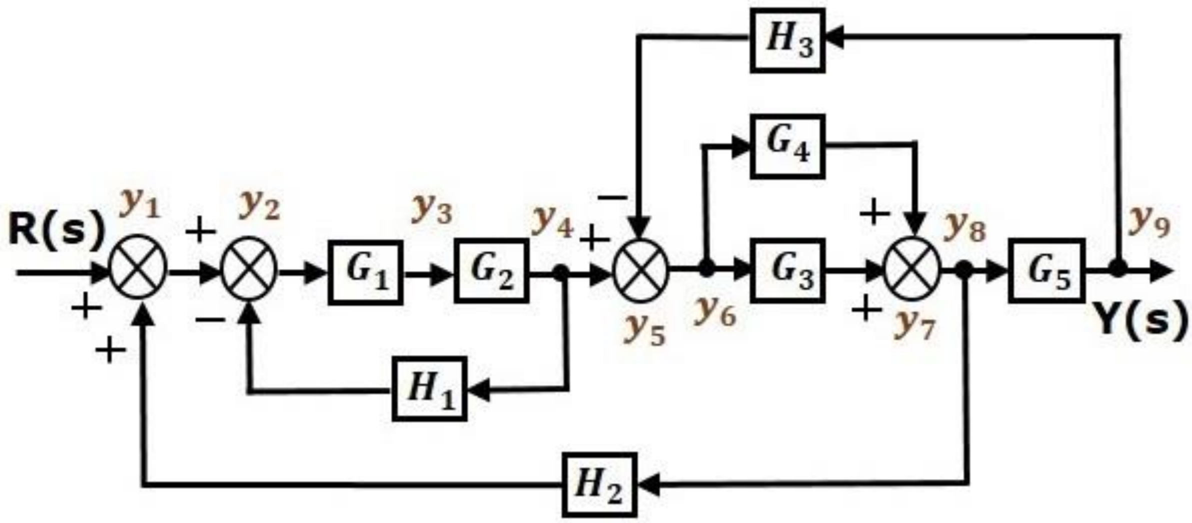
Follow these steps for converting a block diagram into its equivalent signal flow graph.

- Represent all the signals, variables, summing points and take-off points of block diagram as **nodes** in signal flow graph.
- Represent the blocks of block diagram as **branches** in signal flow graph.
- Represent the transfer functions inside the blocks of block diagram as **gains** of the branches in signal flow graph.
- Connect the nodes as per the block diagram. If there is connection between two nodes (but there is no block in between), then represent the gain of the branch as one**. For example**,

between summing points, between summing point and takeoff point, between input and summing point, between take-off point and output.

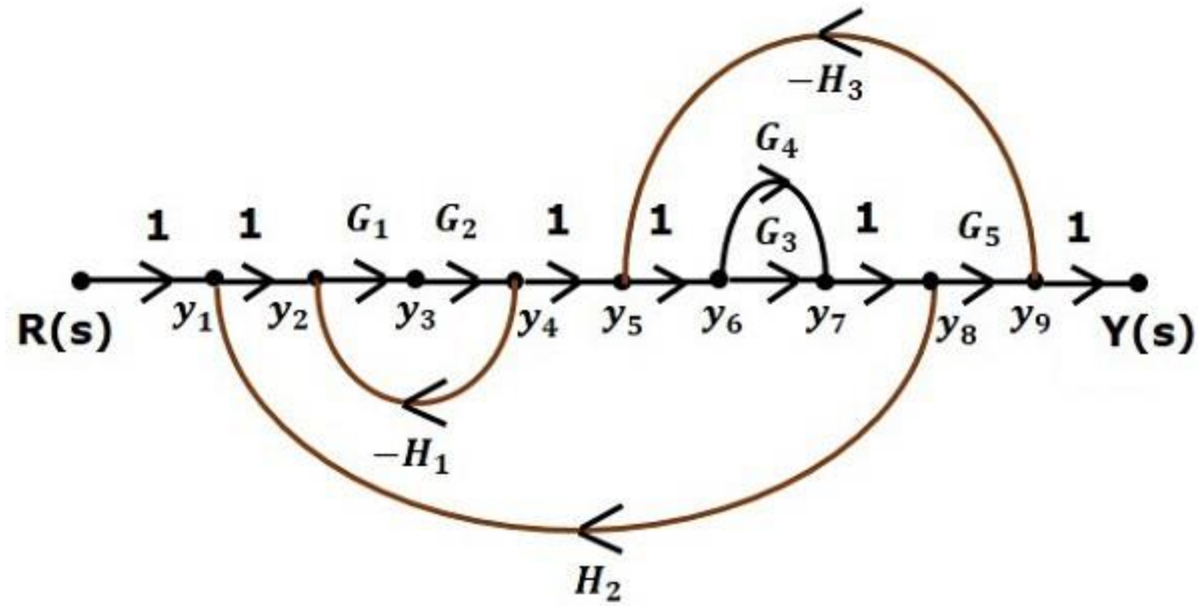
Example

Let us convert the following block diagram into its equivalent signal flow graph.



Represent the input signal R(s)R(s) and output signal C(s)C(s) of block diagram as input node R(s)R(s) and output node C(s)C(s) of signal flow graph.
Just for reference, the remaining nodes (y_1 to y_9) are labelled in the block diagram. There are nine nodes other than input and output nodes. That is four nodes for four summing points, four nodes for four take-off points and one node for the variable between blocks G1G1 and G2G2.

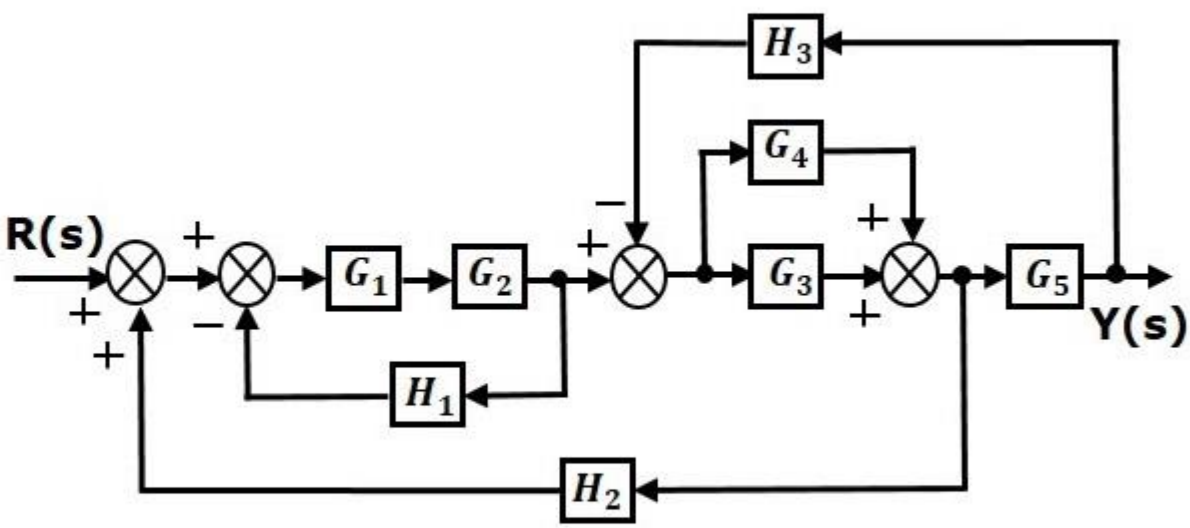
The following figure shows the equivalent signal flow graph.



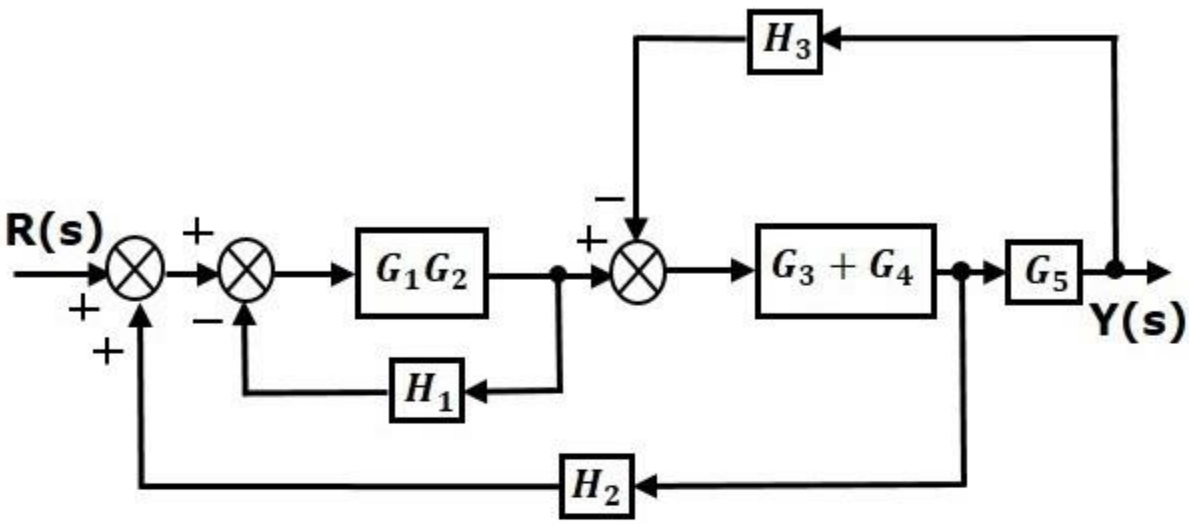
With the help of Mason's gain formula (discussed in the next chapter), you can calculate the transfer function of this signal flow graph. This is the advantage of signal flow graphs. Here, we no need to simplify (reduce) the signal flow graphs for calculating the transfer function.

Q2.

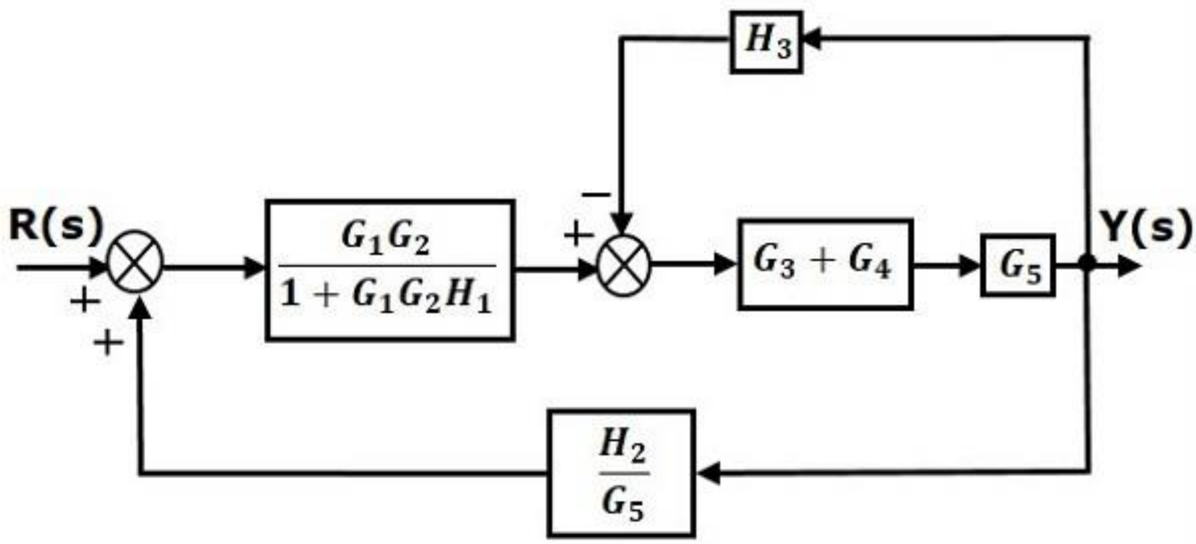
Consider the block diagram shown in the following figure. Let us simplify (reduce) this block diagram using the block diagram reduction rules.



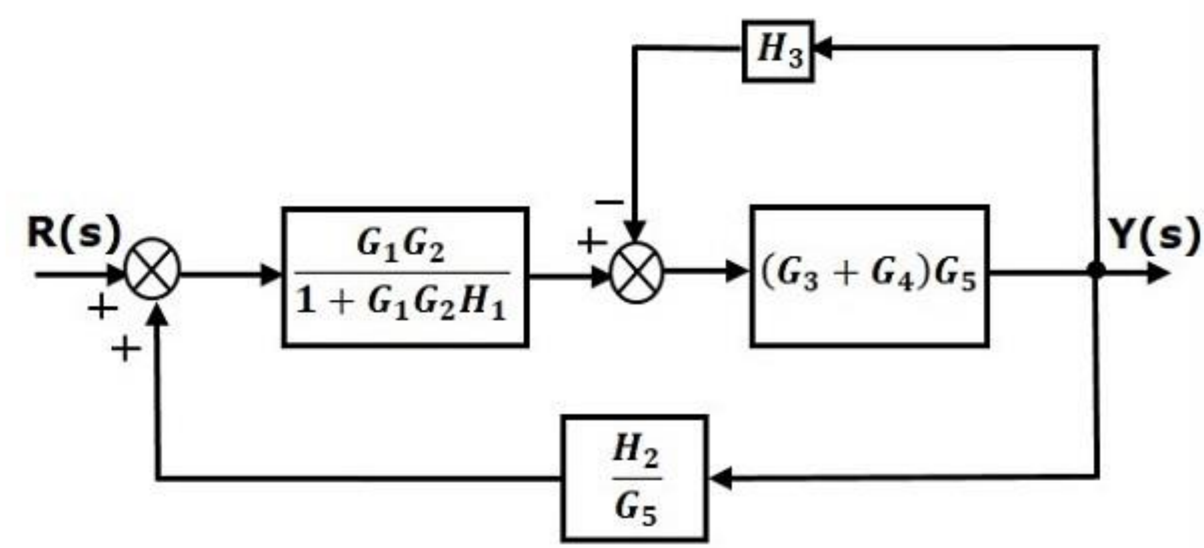
Step 1 − Use Rule 1 for blocks G_1G1 and G_2G2. Use Rule 2 for blocks G_3G3 and G_4G4. The modified block diagram is shown in the following figure.



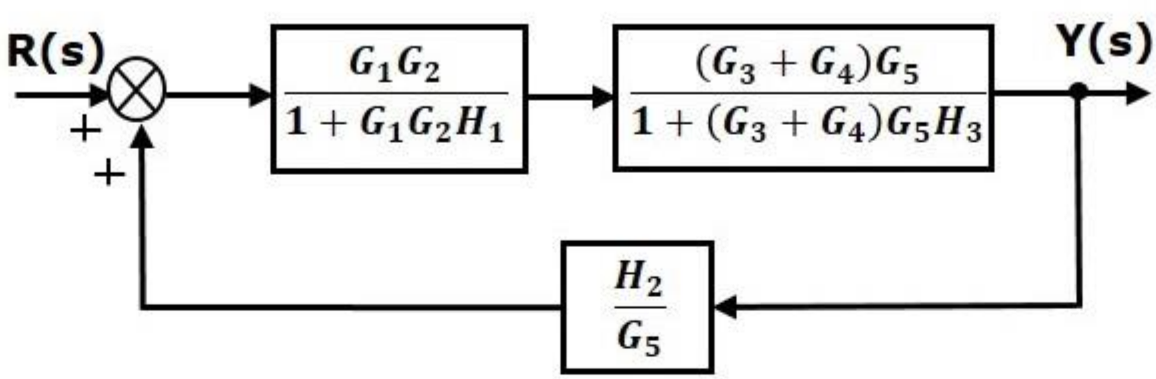
Step 2 − Use Rule 3 for blocks G_1G_2G1G2 and H_1H1. Use Rule 4 for shifting take-off point after the block G_5G5. The modified block diagram is shown in the following figure.



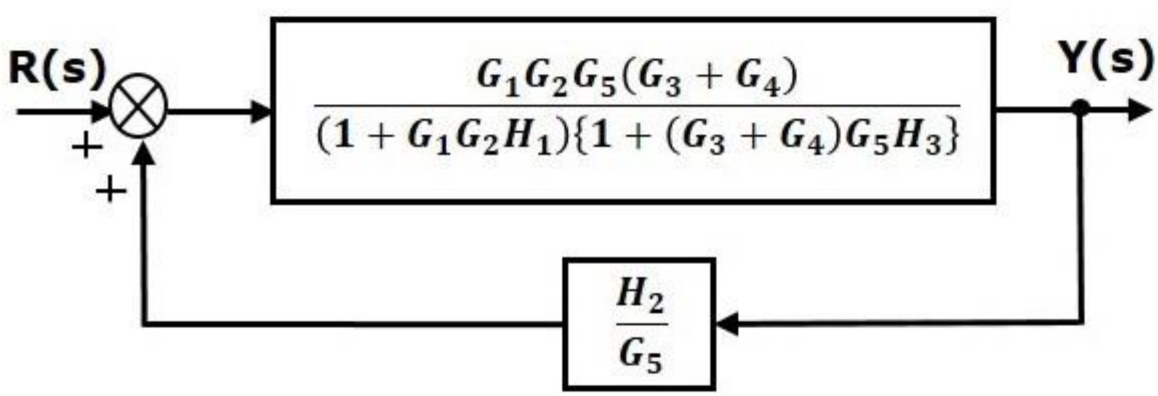
Step 3 − Use Rule 1 for blocks (G_3+G_4)(G3+G4) and G_5G5. The modified block diagram is shown in the following figure.



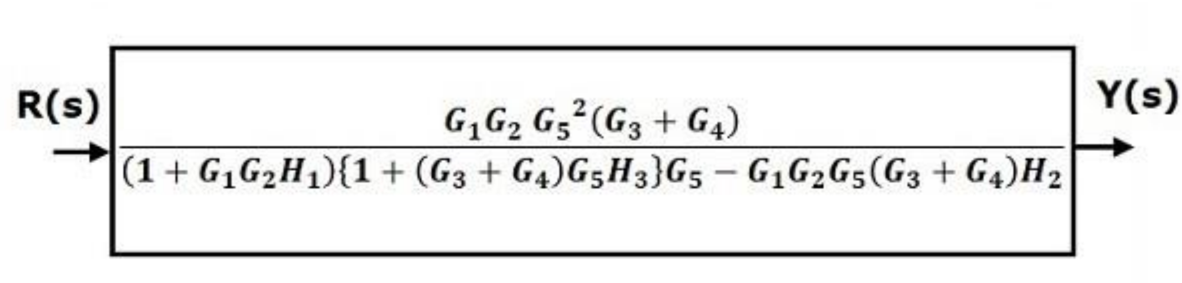
Step 4 − Use Rule 3 for blocks $(G_3+G_4)G_5$(G3+G4)G5 and H_3H3. The modified block diagram is shown in the following figure.



Step 5 − Use Rule 1 for blocks connected in series. The modified block diagram is shown in the following figure.



Step 6 − Use Rule 3 for blocks connected in feedback loop. The modified block diagram is shown in the following figure. This is the simplified block diagram.



Therefore, the transfer function of the system is

$$Y(s)R(s)=G_1G_2G_{25}(G_3+G_4)(1+G_1G_2H_1)\{1+(G_3+G_4)G_5H_3\}G_5-G_1G_2G_5(G_3+G_4)H_2$$

Q3.(a)

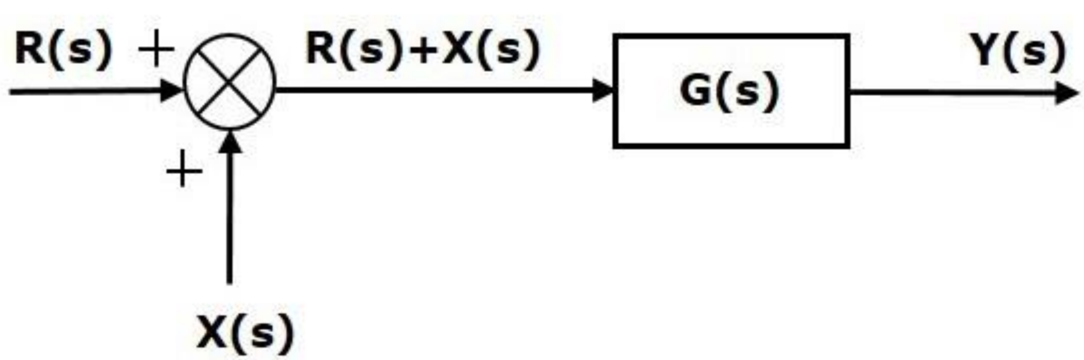
There are two possibilities of shifting summing points with respect to blocks −

- Shifting summing point after the block
- Shifting summing point before the block

Let us now see what kind of arrangements need to be done in the above two cases one by one.

Shifting Summing Point After the Block

Consider the block diagram shown in the following figure. Here, the summing point is present before the block.

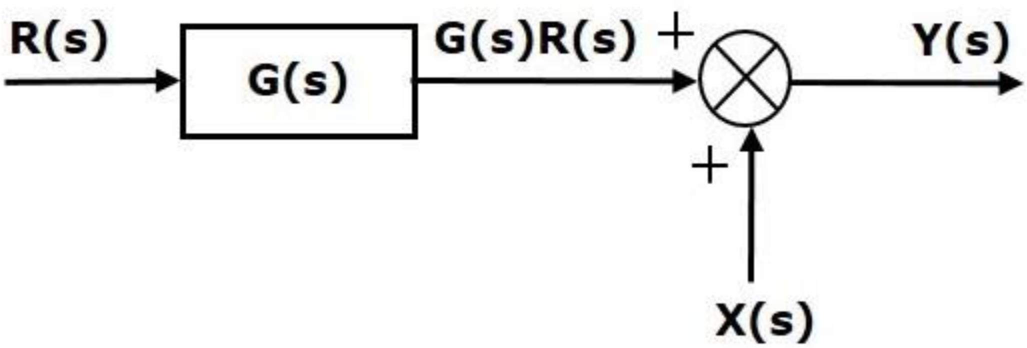


Summing point has two inputs $R(s)$R(s) and $X(s)$X(s). The output of it is $\{R(s)+X(s)\}$\{R(s)+X(s)\}.
So, the input to the block $G(s)$G(s) is $\{R(s)+X(s)\}$\{R(s)+X(s)\} and the output of it is –

$$Y(s)=G(s)\{R(s)+X(s)\}Y(s)=G(s)\{R(s)+X(s)\}$$

$\Rightarrow Y(s)=G(s)R(s)+G(s)X(s) \Rightarrow Y(s)=G(s)R(s)+G(s)X(s)$ **(Equation 1)**

Now, shift the summing point after the block. This block diagram is shown in the following figure.



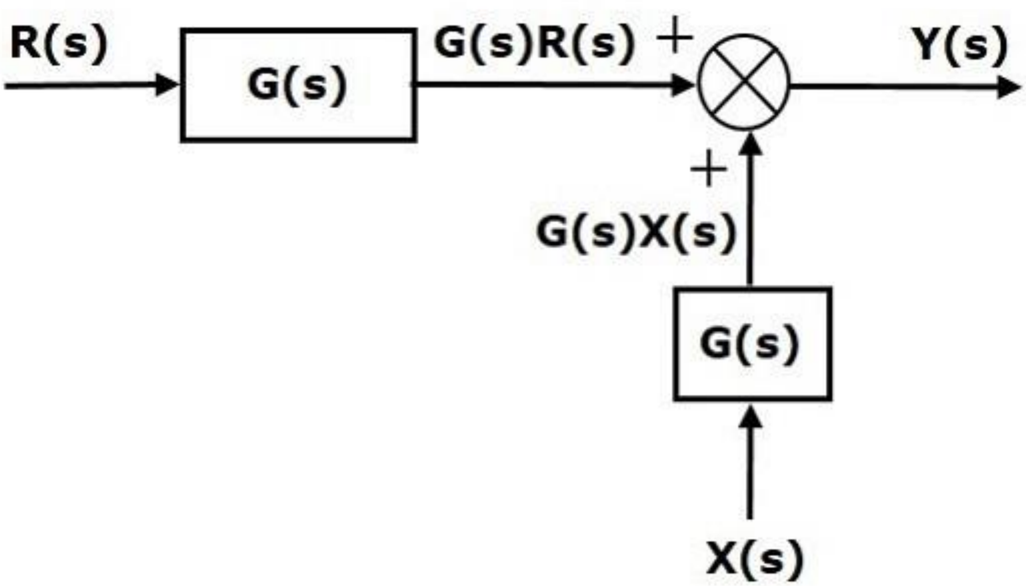
Output of the block $G(s)G(s)$ is $G(s)R(s)G(s)R(s)$.

The output of the summing point is

$Y(s)=G(s)R(s)+X(s)Y(s)=G(s)R(s)+X(s)$ **(Equation 2)**

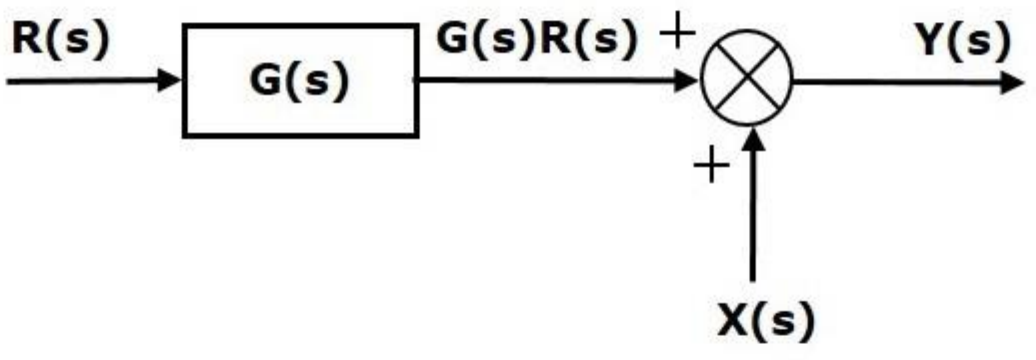
Compare Equation 1 and Equation 2.

The first term '$G(s)R(s)$'$G(s)R(s)$' is same in both the equations. But, there is difference in the second term. In order to get the second term also same, we require one more block $G(s)G(s)$. It is having the input $X(s)X(s)$ and the output of this block is given as input to summing point instead of $X(s)X(s)$. This block diagram is shown in the following figure.



Shifting Summing Point Before the Block

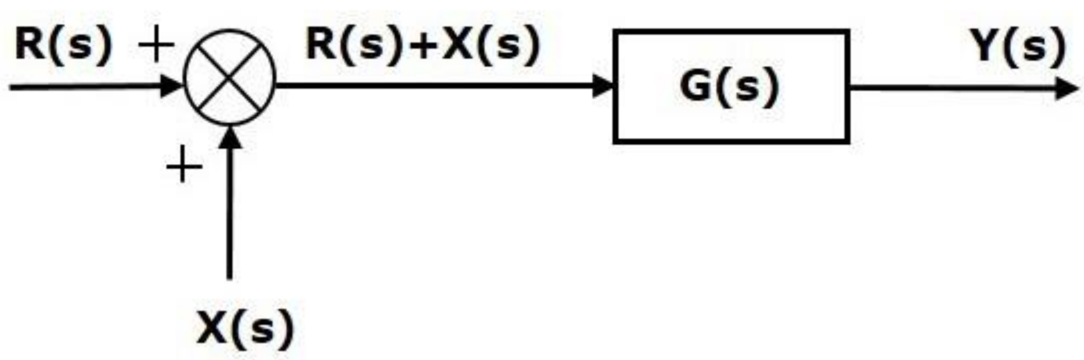
Consider the block diagram shown in the following figure. Here, the summing point is present after the block.



Output of this block diagram is -

$Y(s)=G(s)R(s)+X(s)$Y(s)=G(s)R(s)+X(s) **(Equation 3)**

Now, shift the summing point before the block. This block diagram is shown in the following figure.

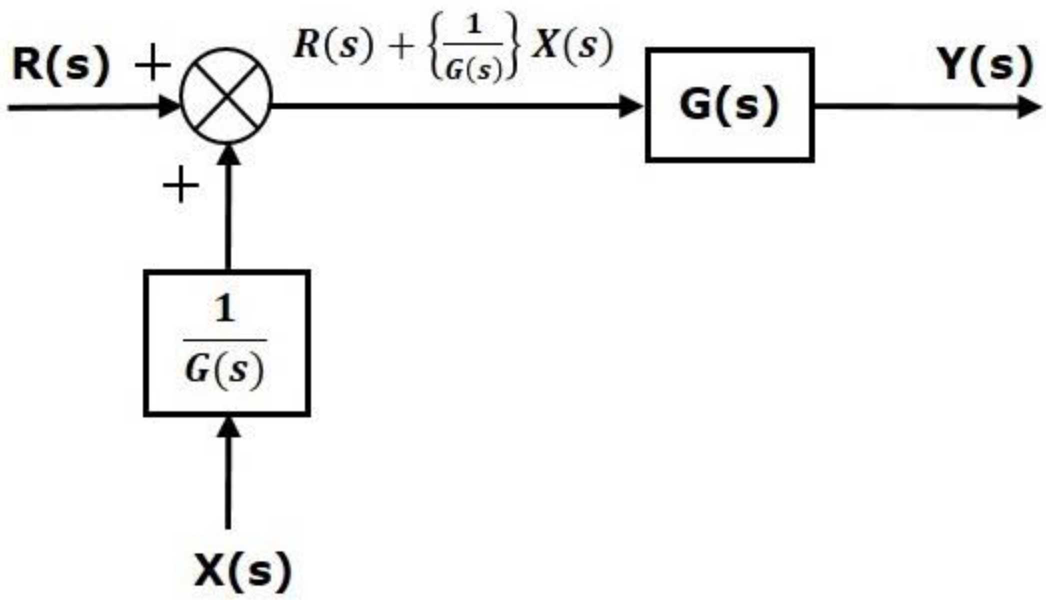


Output of this block diagram is -

$Y(S)=G(s)R(s)+G(s)X(s)$Y(S)=G(s)R(s)+G(s)X(s) **(Equation 4)**

Compare Equation 3 and Equation 4,

The first term '$G(s)R(s)$'G(s)R(s)' is same in both equations. But, there is difference in the second term. In order to get the second term also same, we require one more block $1G(s)$1G(s). It is having the input $X(s)$X(s) and the output of this block is given as input to summing point instead of $X(s)$X(s). This block diagram is shown in the following figure.



Block Diagram Algebra for Take-off Points

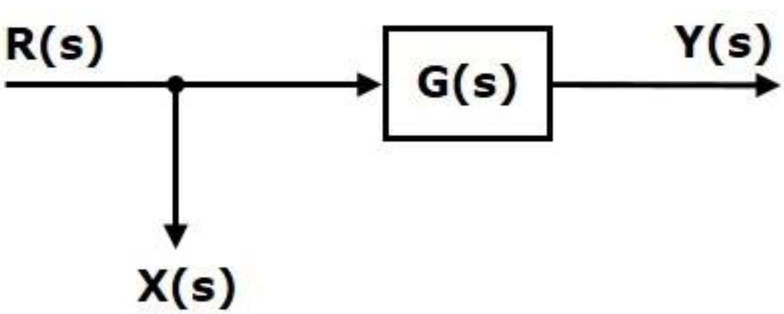
There are two possibilities of shifting the take-off points with respect to blocks −

- Shifting take-off point after the block
- Shifting take-off point before the block

Let us now see what kind of arrangements are to be done in the above two cases, one by one.

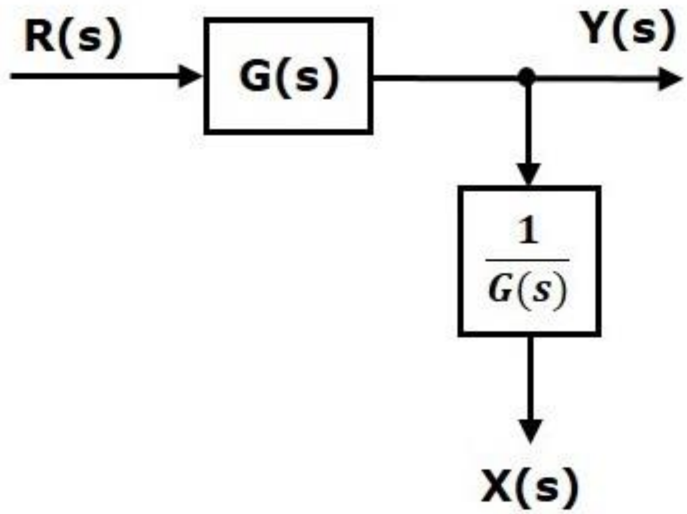
Shifting Take-off Point After the Block

Consider the block diagram shown in the following figure. In this case, the take-off point is present before the block.



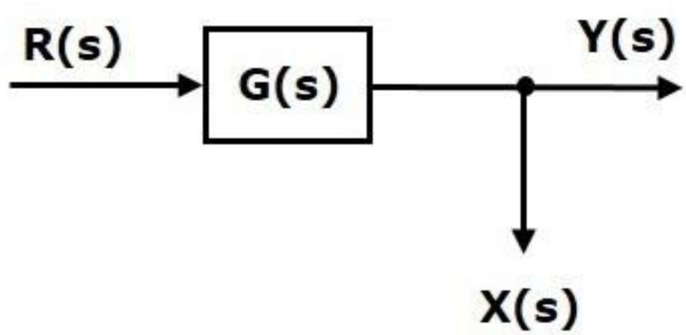
Here, $X(s)=R(s)$X(s)=R(s) and $Y(s)=G(s)R(s)$Y(s)=G(s)R(s)
When you shift the take-off point after the block, the output $Y(s)$Y(s) will be same. But, there is difference in $X(s)$X(s) value. So, in order to get the same $X(s)$X(s) value, we

require one more block 1G(s)1G(s). It is having the input Y(s)Y(s) and the output is X(s)X(s). This block diagram is shown in the following figure.



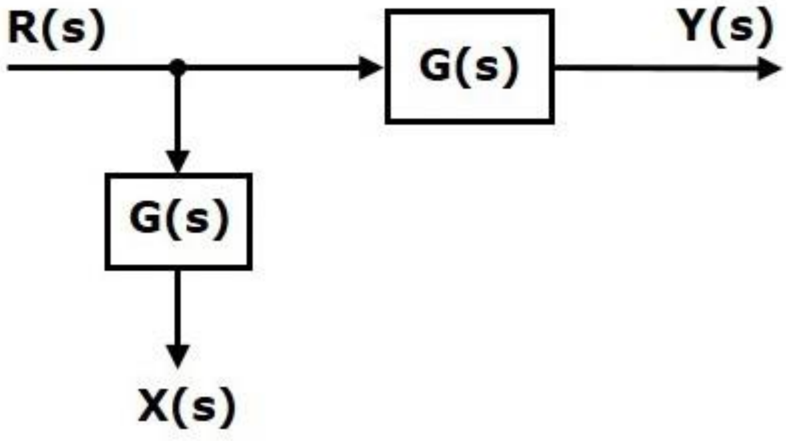
Shifting Take-off Point Before the Block

Consider the block diagram shown in the following figure. Here, the take-off point is present after the block.



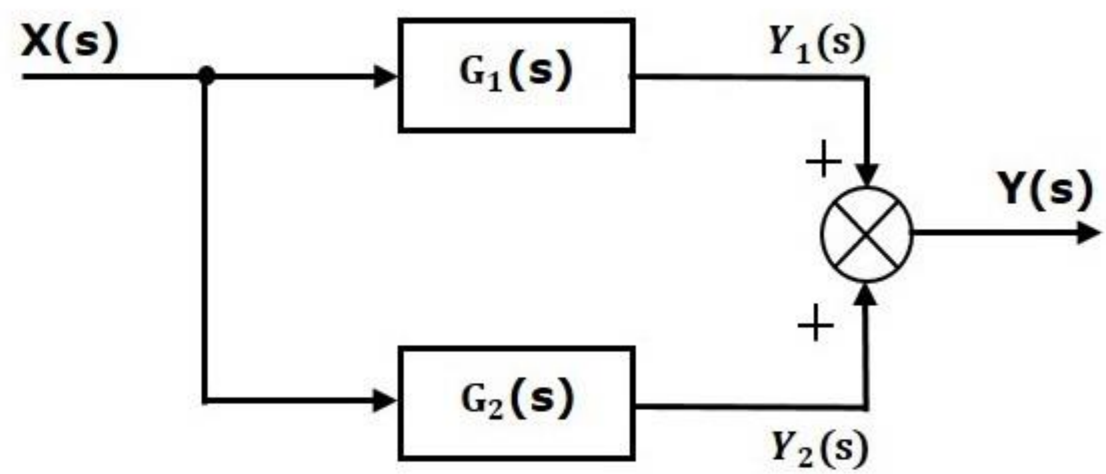
Here, X(s)=Y(s)=G(s)R(s)X(s)=Y(s)=G(s)R(s)

When you shift the take-off point before the block, the output Y(s)Y(s) will be same. But, there is difference in X(s)X(s) value. So, in order to get same X(s)X(s) value, we require one more block G(s)G(s). It is having the input R(s)R(s) and the output is X(s)X(s). This block diagram is shown in the following figure.



Q3.(b)

The blocks which are connected in **parallel** will have the **same input**. In the following figure, two blocks having transfer functions $G_1(s)$G1(s) and $G_2(s)$G2(s) are connected in parallel. The outputs of these two blocks are connected to the summing point.



For this combination, we will get the output $Y(s)$Y(s) as

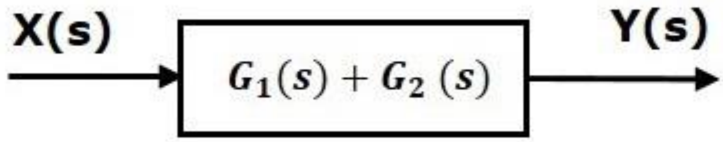
$$Y(s)=Y_1(s)+Y_2(s)$$ Y(s)=Y1(s)+Y2(s)

Where, $Y_1(s)=G_1(s)X(s)$Y1(s)=G1(s)X(s) and $Y_2(s)=G_2(s)X(s)$Y2(s)=G2(s)X(s)

$\Rightarrow Y(s)=G_1(s)X(s)+G_2(s)X(s)=\{G_1(s)+G_2(s)\}X(s)$⇒Y(s)=G1(s)X(s)+G2(s)X(s)={G1(s)+G2(s)}X(s)

Compare this equation with the standard form of the output equation, $Y(s)=G(s)X(s)$Y(s)=G(s)X(s).
Where, $G(s)=G_1(s)+G_2(s)$G(s)=G1(s)+G2(s).

That means we can represent the **parallel connection** of two blocks with a single block. The transfer function of this single block is the **sum of the transfer functions** of those two blocks. The equivalent block diagram is shown below.



Similarly, you can represent parallel connection of 'n' blocks with a single block. The transfer function of this single block is the algebraic sum of the transfer functions of all those 'n' blocks.