Lecture 6

Three Dimensional Transformations:

Methods for geometric transforamtions and object modelling in 3D are extended from 2D methods by including
the considerations for the z coordinate.
Basic geometric transformations are: Translation, Rotation, Scaling

6.1 Basic Transformations

Translation

We translate a 3D point by adding translation distances, tx, ty, and tz, to the original coordinate position (x,y,z):

x' = x + tx, y' = y + ty, z' = z + tz

Alternatively, translation can also be specified by the transformation matrix in the following formula:

x'	1	0	0	t_x	x
y'	0	1	0	t	y
z'	0	0	1	tz	z
1	0	0	0	1	1

Scaling With Respect to the Origin

We scale a 3D object with respect to the origin by setting the scaling factors sx, sy and sz, which are
multiplied to the original vertex coordinate positions (x,y,z):

x' = x * sx, y' = y * sy, z' = z * sz

Alternatively, this scaling can also be specified by the transformation matrix in the following formula:

x'	s_x	0	0	0	x
y'	0	s_y	0	0	y
z'	0	0	sz	0	z
1	0	0	0	1	1

6.2 Scaling with respect to a Selected Fixed Position

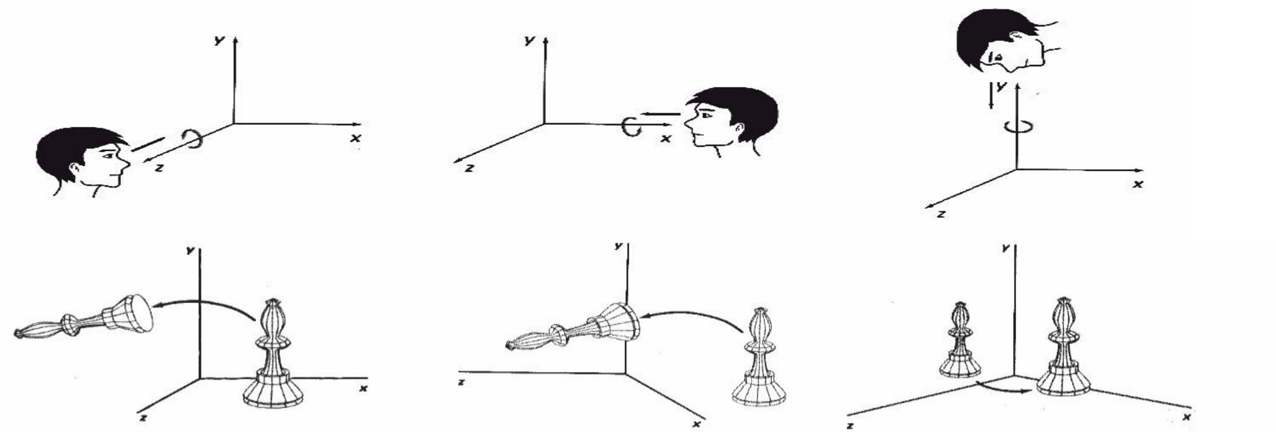
Exercise: What are the steps to perform scaling with respect to a selected fixed position? Check your answer with the text book.

Exercise: Scale a triangle with vertices at original coordinates (10,25,5), (5,10,5), (20,10,10) by sx=1.5, sy=2, and sz=0.5 with respect to the centre of the triangle. For verification, roughly plot the x and y values of the original and resultant triangles, and imagine the locations of z values.

Coordinate-Axes Rotations

A 3D rotation can be specified around any line in space. The easiest rotation axes to handle are the

coordinate axes.



Z-axis rotation: $x' = x \cos\theta - y \sin\theta$

$y' = x \sin\theta + y \cos\theta$, and

$z' = z$

write matrix for z- axis rotation

X-axis rotation:

$y' = y \cos\theta - z \sin\theta$,

$z' = y \sin\theta + z \cos\theta$, and

$x' = x$

write matrix for x- axis rotation

Y-axis rotation:

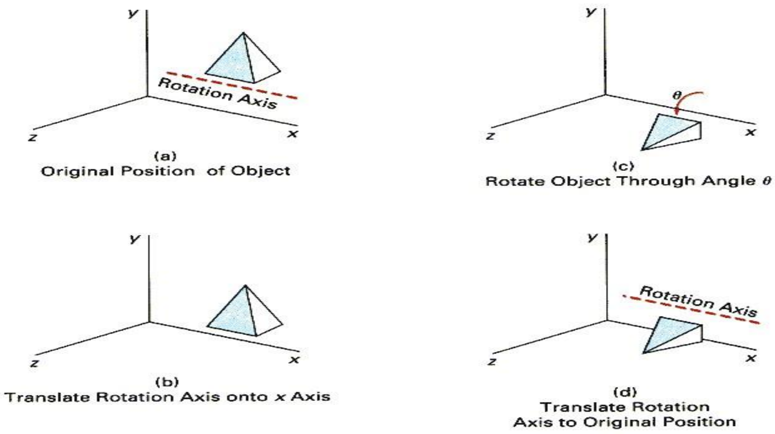
$z' = z \cos\theta - x \sin\theta$

$x' = z \sin\theta + x \cos\theta$, and

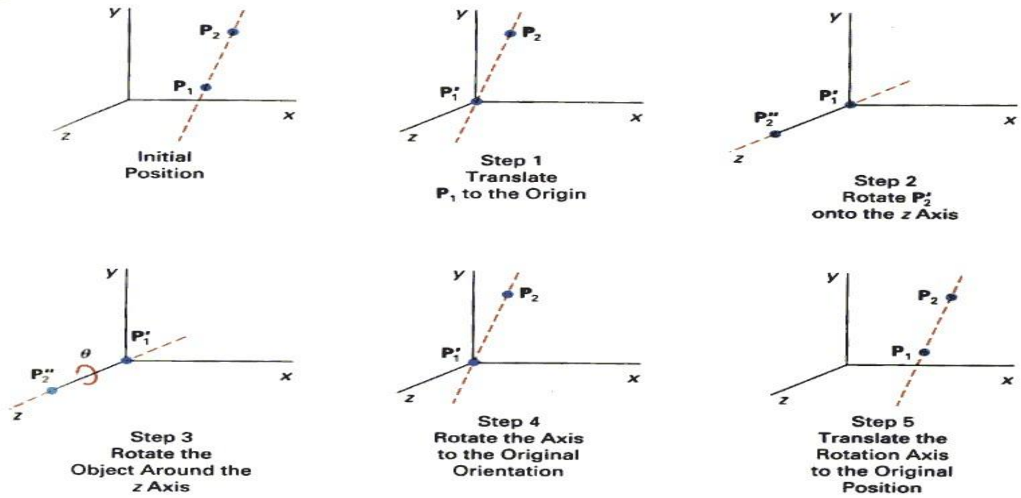
$y' = y$

write matrix for y- axis rotation

3D Rotations About an Axis Which is Parallel to an Axis



Step 1. Translate the object so that the rotation axis coincides with the parallel coordinate axis.
Step 2. Perform the specified rotation about that axis.
Step 3. Translate the object so that the rotation axis is moved back to its original position.
General 3D Rotations



Step 1. Translate the object so that the rotation axis passes through the coordinate origin.
Step 2. Rotate the object so that the axis of rotation coincides with one of the coordinate axes.
Step 3. Perform the specified rotation about that coordinate axis.
Step 4. Rotate the object so that the rotation axis is brought back to its original orientation.
Step 5. Translate the object so that the rotation axis is brought back to its original position.

6.3 Three-Dimensional Viewing

Viewing in 3D involves the following considerations:

- We can view an object from any spatial position, eg. In front of an object, Behind the object, In the middle of a group of objects, Inside an object, etc.
- 3D descriptions of objects must be projected onto the flat viewing surface of the output device.
- The clipping boundaries enclose a volume of space

6.4 Viewing Pipeline

Modelling Coordinates

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Modelling Transformations

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World Coordinates

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Viewing Transformation

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Viewing Coordinates

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Projection Transformation

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Projection Coordinates

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Workstation Transformation

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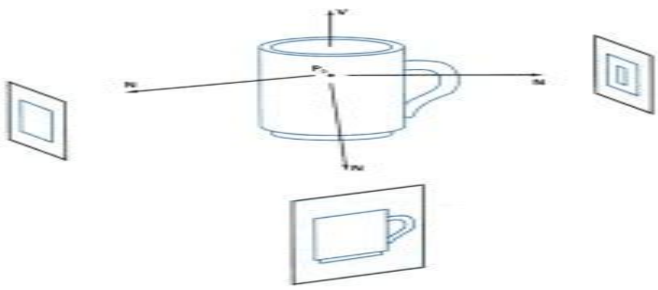
Device Coordinates

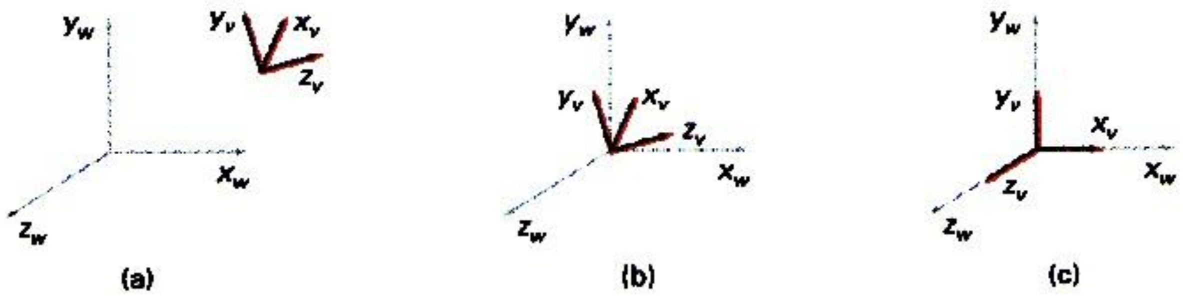
Modelling Transformation and Viewing Transformation can be done by 3D transformations. The viewing-coordinate system is used in graphics packages as a reference for specifying the observer viewing position and the position of the projection plane. Projection operations convert the viewing-coordinate description (3D) to coordinate positions on the projection plane (2D). (Usually combined with clipping, visual-surface identification, and surface-rendering)Workstation transformation maps the coordinate positions on the projection plane to the output device.



6.5 Viewing Transformation

Conversion of objection descriptions from world to viewing coordinates is equivalent to a transformation that superimposes the viewing reference frame onto the world frame using the basic
geometric translate-rotate operations:

1. Translate the view reference point to the origin of the world-coordinate system.
2. Apply rotations to align the xv, yv, and zv axes (viewing coordinate system) with the world xw, yw,zw axes, respectively.



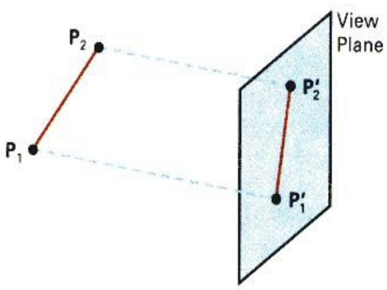
6.6 Projections

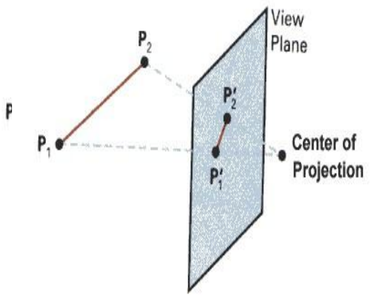
Projection operations convert the viewing-coordinate description (3D) to coordinate positions on the projection plane (2D). There are 2 basic projection methods:

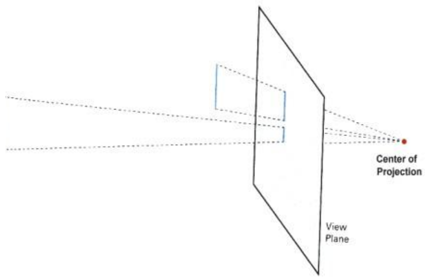
1. Parallel Projection transforms object positions to the view plane along parallel lines.

A parallel projection preserves relative proportions of objects. Accurate views of the various sides ofan object are obtained with a parallel projection. But not a realistic representation

2. Perspective Projection transforms object positions to the view plane while converging to a center
point of projection. Perspective projection produces realistic views but does not preserve relative proportions. Projections of distant objects are smaller than the projections of objects of the same size that are closer to the
projection plane.







6.6.1 Parallel Projection

Classification:

Orthographic Parallel Projection and Oblique Projection:

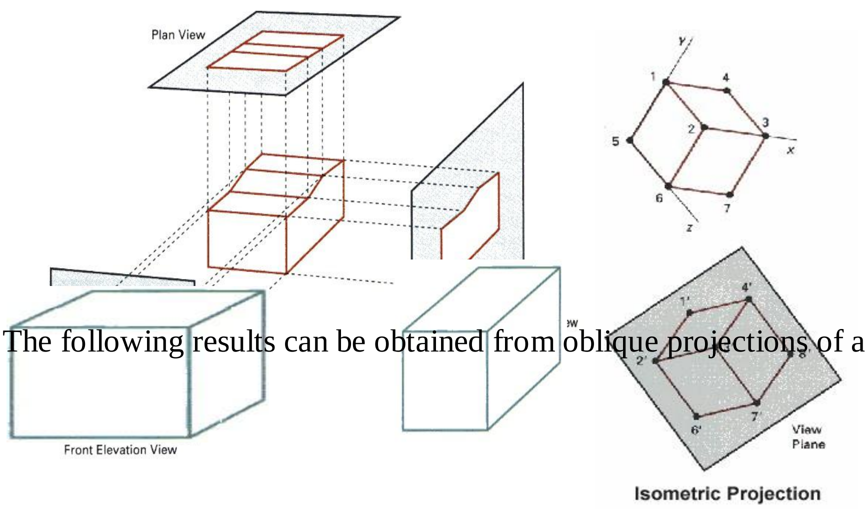
Orthographic parallel projections are done by projecting points along parallel lines that are perpendicular to the projection plane.

Oblique projections are obtained by projecting along parallel lines that are NOT perpendicular

to

the projection plane.Some special Orthographic Parallel Projections involve Plan View

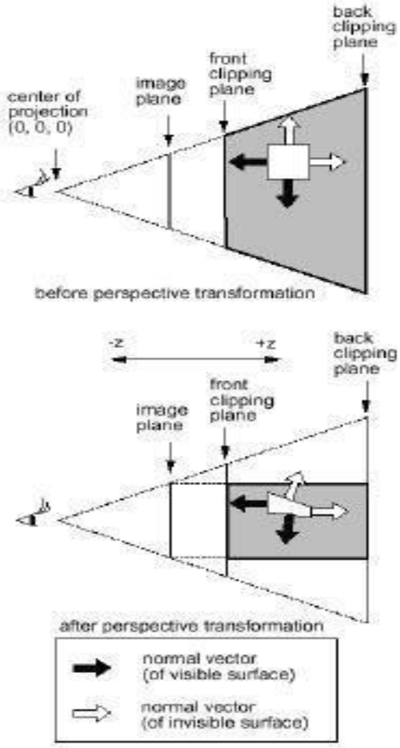
(Top projection), Side Elevations, and Isometric Projection:

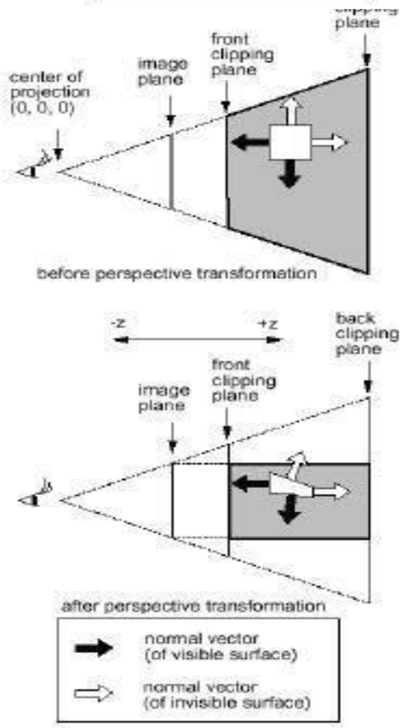


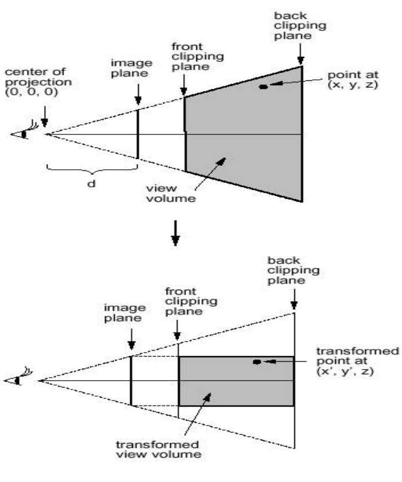
The following results can be obtained from oblique projections of a cube:

6.6.2 Perspective Projection

In Perspective projection all lines converge to a point called projection reference point or center of projection

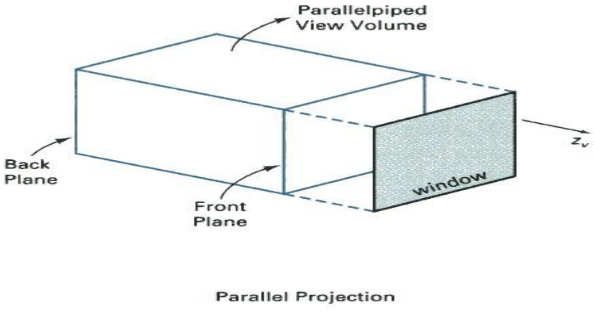


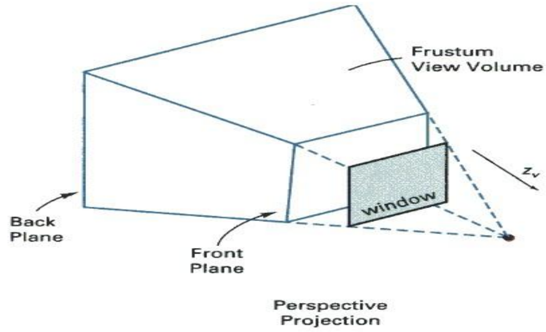




6.7 View Volumes

View window - A rectangular area in the view plane which controls how much of the scene is viewed.The edges of the view window are parallel to the xv and yv viewing axes. View volume - formed by the view window and the type of projection to be used. Only those objects within the view volume will appear in the generated display. Hence a view volume is bounded by 6 planes => rectangular parallelepiped or a frustum, for parallel projection and perspective projection respectively.





6.8 Clipping

The purpose of 3D clipping is to identify and save all surface segments within the view volume for display on the output device. All parts of objects that are outside the view volume are discarded. Thus the computing time is saved. 3D clipping is based on 2D clipping.