Forecasting Approaches

Quantitative forecasts
Forecasts that employ mathematical modeling to forecast demand.

Qualitative forecasts
Forecasts that incorporate such factors as the decision maker's intuition, emotions, personal experiences, and value system.

There are two general approaches to forecasting, just as there are two ways to tackle all decision modeling. One is a quantitative analysis; the other is a qualitative approach. **Quantitative forecasts** use a variety of mathematical models that rely on historical data and/or associative variables to forecast demand. Subjective or **qualitative forecasts** incorporate such factors as the decision maker's intuition, emotions, personal experiences, and value system in reaching a forecast. Some firms use one approach and some use the other. In practice, a combination of the two is usually most effective.

Overview of Qualitative Methods

In this section, we consider four different *qualitative* forecasting techniques:

Jury of executive opinion
A forecasting technique that uses the opinion of a small group of high-level managers to form a group estimate of demand.

Delphi method
A forecasting technique using a group process that allows experts to make forecasts.

LO 4.2 *Explain* when to use each of the four qualitative models

Sales force composite
A forecasting technique based on salespersons' estimates of expected sales.

Market survey
A forecasting method that solicits input from customers or potential customers regarding future purchasing plans.

1. **Jury of executive opinion:** Under this method, the opinions of a group of high-level experts or managers, often in combination with statistical models, are pooled to arrive at a group estimate of demand. Bristol-Myers Squibb Company, for example, uses 220 well-known research scientists as its jury of executive opinion to get a grasp on future trends in the world of medical research.
2. **Delphi method:** There are three different types of participants in the Delphi method: decision makers, staff personnel, and respondents. Decision makers usually consist of a group of 5 to 10 experts who will be making the actual forecast. Staff personnel assist decision makers by preparing, distributing, collecting, and summarizing a series of questionnaires and survey results. The respondents are a group of people, often located in different places, whose judgments are valued. This group provides inputs to the decision makers before the forecast is made.

   The state of Alaska, for example, has used the Delphi method to develop its long-range economic forecast. A large part of the state's budget is derived from the million-plus barrels of oil pumped daily through a pipeline at Prudhoe Bay. The large Delphi panel of experts had to represent all groups and opinions in the state and all geographic areas.
3. **Sales force composite:** In this approach, each salesperson estimates what sales will be in his or her region. These forecasts are then reviewed to ensure that they are realistic. Then they are combined at the district and national levels to reach an overall forecast. A variation of this approach occurs at Lexus, where every quarter Lexus dealers have a "make meeting." At this meeting, they talk about what is selling, in what colors, and with what options, so the factory knows what to build.
4. **Market survey:** This method solicits input from customers or potential customers regarding future purchasing plans. It can help not only in preparing a forecast but also in improving product design and planning for new products. The consumer market survey and sales force composite methods can, however, suffer from overly optimistic forecasts that arise from customer input.

Overview of Quantitative Methods[1]

Five quantitative forecasting methods, all of which use historical data, are described in this chapter. They fall into two categories:

1. Naive approach 2. Moving averages 3. Exponential smoothing 4. Trend projection	} **Time-series models**
5. Linear regression	} **Associative model**

Time-Series Models Time-series models predict on the assumption that the future is a function of the past. In other words, they look at what has happened over a period of time and use a series of past data to make a forecast. If we are predicting sales of lawn mowers, we use the past sales for lawn mowers to make the forecasts.

Time series
A forecasting technique that uses a series of past data points to make a forecast.

Associative Models Associative models, such as linear regression, incorporate the variables or factors that might influence the quantity being forecast. For example, an associative model for lawn mower sales might use factors such as new housing starts, advertising budget, and competitors' prices.

Time-Series Forecasting

STUDENT TIP
Here is the meat of this chapter. We now show you a wide variety of models that use time-series data.

A time series is based on a sequence of evenly spaced (weekly, monthly, quarterly, and so on) data points. Examples include weekly sales of Nike Air Jordans, quarterly earnings reports of Microsoft stock, daily shipments of Coors beer, and annual consumer price indices. Forecasting time-series data implies that future values are predicted *only* from past values and that other variables, no matter how potentially valuable, may be ignored.

Decomposition of a Time Series

Analyzing time series means breaking down past data into components and then projecting them forward. A time series has four components:

1. *Trend* is the gradual upward or downward movement of the data over time. Changes in income, population, age distribution, or cultural views may account for movement in trend.
2. *Seasonality* is a data pattern that repeats itself after a period of days, weeks, months, or quarters. There are six common seasonality patterns:

STUDENT TIP
The peak "seasons" for sales of Frito-Lay chips are the Super Bowl, Memorial Day, Labor Day, and the Fourth of July.

PERIOD LENGTH	"SEASON" LENGTH	NUMBER OF "SEASONS" IN PATTERN
Week	Day	7
Month	Week	$4–4\frac{1}{2}$
Month	Day	28–31
Year	Quarter	4
Year	Month	12
Year	Week	52

Restaurants and barber shops, for example, experience weekly seasons, with Saturday being the peak of business. Beer distributors forecast yearly patterns, with monthly seasons. Three "seasons"—May, July, and September—each contain a big beer-drinking holiday.

3. *Cycles* are patterns in the data that occur every several years. They are usually tied into the business cycle and are of major importance in short-term business analysis and planning. Predicting business cycles is difficult because they may be affected by political events or by international turmoil.
4. *Random variations* are "blips" in the data caused by chance and unusual situations. They follow no discernible pattern, so they cannot be predicted.

OM in Action Forecasting the Next Fog

India suffers from thick fog in its north-western States, starting from December to February every year. The threat of fog has drastically increased in recent years, due to heavy pollution and land use changes. Most areas in these states woke up to a hazy morning experiencing the dense fog cover of winter season. The visibility becomes very low in some regions impacting flight operations and vehicular movement and economic activities across these States. Ministry of Earth Sciences (MoES) has launched new model of fog forecasting at the Indira Gandhi International (IGI) airport in December 2016. This will be a part of a three-month long observational campaign by the Pune-based Indian Institute of Tropical Meteorology (IITM). IITM has highlighted that the campaign will offer scientists and researchers, an opportunity to experiment their model on numerous temporal and spatial scales at the IGI airport. IGI airport encounters heavy loss of working hours each year, because of fog. Testing the new fog forecasting model here will enormously help the IITM to devise an almost accurate mechanism to handle the fog next winter. IITM will use advanced forecasting methods to warn people to reduce the impact of the fog on aviation, rail and road transport movements. The Indian Meteorological Department (IMD) has been issuing weather reports since long, but the new drive by the IITM will further improve capability in predicting the movement of fog.

Adapted from The Pioneer Columnists, 'Forecasting the Next Fog', http://www.dailypioneer.com/columnists/edit/forecasting-the-next-fog.html , as accessed on February 2, 2017 at 4.19pm

LO 4.3 *Apply* the naive, moving-average, exponential smoothing, and trend methods

Figure 4.1 illustrates a demand over a 4-year period. It shows the average, trend, seasonal components, and random variations around the demand curve. The average demand is the sum of the demand for each period divided by the number of data periods.

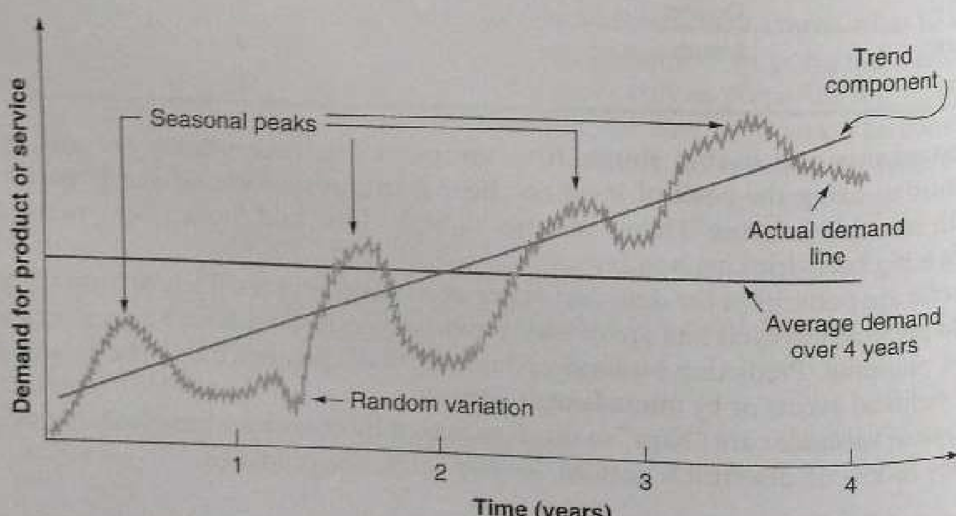
Naive Approach

The simplest way to forecast is to assume that demand in the next period will be equal to demand in the most recent period. In other words, if sales of a product—say, Nokia cell phones—were 68 units in January, we can forecast that February's sales will also be 68 phones. Does this make any sense? It turns out that for some product lines, this **naive approach** is the most cost-effective and efficient objective forecasting model. At least it provides a starting point against which more sophisticated models that follow can be compared.

Naive approach
A forecasting technique that assumes that demand in the next period is equal to demand in the most recent period.

Figure **4.1**

Demand Charted over 4 Years, with a Growth Trend and Seasonality Indicated



STUDENT TIP
Forecasting is easy when demand is stable. But with trend, seasonality, and cycles considered, the job is a lot more interesting.

Moving Averages

A **moving-average** forecast uses a number of historical actual data values to generate a forecast. Moving averages are useful *if we can assume that market demands will stay fairly steady over time*. A 4-month moving average is found by simply summing the demand during the past 4 months and dividing by 4. With each passing month, the most recent month's data are added to the sum of the previous 3 months' data, and the earliest month is dropped. This practice tends to smooth out short-term irregularities in the data series.

Moving averages
A forecasting method that uses an average of the n most recent periods of data to forecast the next period.

Mathematically, the simple moving average (which serves as an estimate of the next period's demand) is expressed as:

$$\text{Moving average} = \frac{\sum \text{demand in previous } n \text{ periods}}{n} \tag{4-1}$$

where n is the number of periods in the moving average—for example, 4, 5, or 6 months, respectively, for a 4-, 5-, or 6-period moving average.

Example 1 shows how moving averages are calculated.

Example 1

DETERMINING THE MOVING AVERAGE

Donna's Garden Supply wants a 3-month moving-average forecast, including a forecast for next January, for shed sales.

APPROACH ▶ Storage shed sales are shown in the middle column of the following table. A 3-month moving average appears on the right.

MONTH	ACTUAL SHED SALES	3-MONTH MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	$(10 + 12 + 13)/3 = 11\frac{2}{3}$
May	19	$(12 + 13 + 16)/3 = 13\frac{2}{3}$
June	23	$(13 + 16 + 19)/3 = 16$
July	26	$(16 + 19 + 23)/3 = 19\frac{1}{3}$
August	30	$(19 + 23 + 26)/3 = 22\frac{2}{3}$
September	28	$(23 + 26 + 30)/3 = 26\frac{1}{3}$
October	18	$(26 + 30 + 28)/3 = 28$
November	16	$(30 + 28 + 18)/3 = 25\frac{1}{3}$
December	14	$(28 + 18 + 16)/3 = 20\frac{2}{3}$

SOLUTION ▶ The forecast for December is $20\frac{2}{3}$. To project the demand for sheds in the coming January, we sum the October, November, and December sales and divide by 3: January forecast $= (18 + 16 + 14)/3 = 16$.

INSIGHT ▶ Management now has a forecast that averages sales for the last 3 months. It is easy to use and understand.

LEARNING EXERCISE ▶ If actual sales in December were 18 (rather than 14), what is the new January forecast? [Answer: $17\frac{1}{3}$.]

RELATED PROBLEMS ▶ 4.1a, 4.2b, 4.5a, 4.6, 4.8a, b, 4.10a, 4.13b, 4.15, 4.33

When a detectable trend or pattern is present, *weights* can be used to place more emphasis on recent values. This practice makes forecasting techniques more responsive to changes because more recent periods may be more heavily weighted. Choice of weights is somewhat arbitrary because there is no set formula to determine them. Therefore, deciding which weights to use requires some experience. For example, if the latest month or period is weighted too heavily, the forecast may reflect a large unusual change in the demand or sales pattern too quickly.

A weighted moving average may be expressed mathematically as:

$$\text{Weighted moving average} = \frac{\sum((\text{Weight for period } n)(\text{Demand in period } n))}{\sum \text{Weights}} \tag{4-2}$$

Example 2 shows how to calculate a weighted moving average.

Example 2

DETERMINING THE WEIGHTED MOVING AVERAGE

Donna's Garden Supply (see Example 1) wants to forecast storage shed sales by weighting the past 3 months, with more weight given to recent data to make them more significant.

APPROACH ▶ Assign more weight to recent data, as follows:

WEIGHTS APPLIED	PERIOD
3	Last month
2	Two months ago
1	Three months ago
6	Sum of weights

$$\text{Forecast for this month} = \frac{3 \times \text{Sales last mo.} + 2 \times \text{Sales 2 mos. ago} + 1 \times \text{Sales 3 mos. ago}}{\text{Sum of the weights}}$$

SOLUTION ▶ The results of this weighted-average forecast are as follows:

MONTH	ACTUAL SHED SALES	3-MONTH WEIGHTED MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	$[(3 \times 13) + (2 \times 12) + (10)]/6 = 12\frac{1}{6}$
May	19	$[(3 \times 16) + (2 \times 13) + (12)]/6 = 14\frac{1}{3}$
June	23	$[(3 \times 19) + (2 \times 16) + (13)]/6 = 17$
July	26	$[(3 \times 23) + (2 \times 19) + (16)]/6 = 20\frac{1}{2}$
August	30	$[(3 \times 26) + (2 \times 23) + (19)]/6 = 23\frac{5}{6}$
September	28	$[(3 \times 30) + (2 \times 26) + (23)]/6 = 27\frac{1}{2}$
October	18	$[(3 \times 28) + (2 \times 30) + (26)]/6 = 28\frac{1}{3}$
November	16	$[(3 \times 18) + (2 \times 28) + (30)]/6 = 23\frac{1}{3}$
December	14	$[(3 \times 16) + (2 \times 18) + (28)]/6 = 18\frac{2}{3}$

The forecast for January is $15\frac{1}{3}$. Do you see how this number is computed?

INSIGHT ▶ In this particular forecasting situation, you can see that more heavily weighting the latest month provides a more accurate projection.

LEARNING EXERCISE ▶ If the assigned weights were 0.50, 0.33, and 0.17 (instead of 3, 2, and 1), what is the forecast for January's weighted moving average? Why? [Answer: There is no change. These are the same *relative* weights. Note that $\sum$ weights = 1 now, so there is no need for a denominator. When the weights sum to 1, calculations tend to be simpler.]

RELATED PROBLEMS ▶ 4.1b, 4.2c, 4.5c, 4.6, 4.7, 4.10b

Both simple and weighted moving averages are effective in smoothing out sudden fluctuations in the demand pattern to provide stable estimates. Moving averages do, however, present three problems:

1. Increasing the size of n (the number of periods averaged) does smooth out fluctuations better, but it makes the method less sensitive to changes in the data.
2. Moving averages cannot pick up trends very well. Because they are averages, they will always stay within past levels and will not predict changes to either higher or lower levels. That is, they *lag* the actual values.
3. Moving averages require extensive records of past data.

Figure 4.2, a plot of the data in Examples 1 and 2, illustrates the lag effect of the moving-average models. Note that both the moving-average and weighted-moving-average lines lag the actual demand. The weighted moving average, however, usually reacts more quickly to demand changes. Even in periods of downturn (see November and December), it more closely tracks the demand.

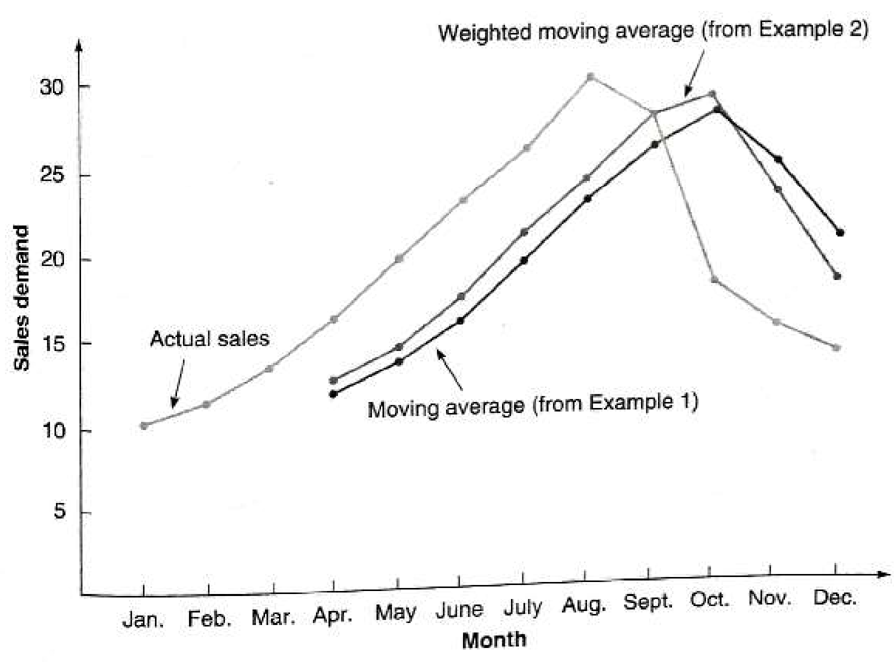


Figure **4.2**

Actual Demand vs. Moving-Average and Weighted-Moving-Average Methods for Donna's Garden Supply

STUDENT TIP

Moving-average methods always lag behind when there is a trend present, as shown by the blue line (actual sales) for January through August.

Exponential Smoothing

Exponential smoothing
A weighted-moving-average forecasting technique in which data points are weighted by an exponential function.

Smoothing constant
The weighting factor used in an exponential smoothing forecast, a number greater than or equal to 0 and less than or equal to 1.

Exponential smoothing is another weighted-moving-average forecasting method. It involves very *little* record keeping of past data and is fairly easy to use. The basic exponential smoothing formula can be shown as follows:

$$\text{New forecast} = \text{Last period's forecast} + \alpha\,(\text{Last period's actual demand} - \text{Last period's forecast}) \quad (4\text{-}3)$$

where α is a weight, or **smoothing constant**, chosen by the forecaster, that has a value greater than or equal to 0 and less than or equal to 1. Equation (4-3) can also be written mathematically as:

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1}) \quad (4\text{-}4)$$

where
F_t = new forecast
F_{t-1} = previous period's forecast
α = smoothing (or weighting) constant ($0 \le \alpha \le 1$)
A_{t-1} = previous period's actual demand

The concept is not complex. The latest estimate of demand is equal to the old forecast adjusted by a fraction of the difference between the last period's actual demand and last period's forecast. Example 3 shows how to use exponential smoothing to derive a forecast.

The *smoothing constant*, α, is generally in the range from .05 to .50 for business applications. It can be changed to give more weight to recent data (when α is high) or more weight to past data (when α is low). When α reaches the extreme of 1.0, then in Equation (4-4), $F_t = 1.0A_{t-1}$. All the older values drop out, and the forecast becomes identical to the naive model mentioned earlier in this chapter. That is, the forecast for the next period is just the same as this period's demand.

Example 3

DETERMINING A FORECAST VIA EXPONENTIAL SMOOTHING

In January, a car dealer predicted February demand for 142 Ford Mustangs. Actual February demand was 153 autos. Using a smoothing constant chosen by management of $\alpha = .20$, the dealer wants to forecast March demand using the exponential smoothing model.

APPROACH ▶ The exponential smoothing model in Equations (4-3) and (4-4) can be applied.

SOLUTION ▶ Substituting the sample data into the formula, we obtain:

$$\text{New forecast (for March demand)} = 142 + .2(153 - 142) = 142 + 2.2$$
$$= 144.2$$

Thus, the March demand forecast for Ford Mustangs is rounded to 144.

INSIGHT ▶ Using just two pieces of data, the forecast and the actual demand, plus a smoothing constant, we developed a forecast of 144 Ford Mustangs for March.

LEARNING EXERCISE ▶ If the smoothing constant is changed to .30, what is the new forecast? [Answer: 145.3]

RELATED PROBLEMS ▶ 4.1c, 4.3, 4.4, 4.5d, 4.6, 4.9d, 4.11, 4.12, 4.13a, 4.17, 4.18, 4.31, 4.33, 4.34

The following table helps illustrate this concept. For example, when $\alpha = .5$, we can see that the new forecast is based almost entirely on demand in the last three or four periods. When $\alpha = .1$, the forecast places little weight on recent demand and takes many periods (about 19) of historical values into account.

	WEIGHT ASSIGNED TO				
SMOOTHING CONSTANT	MOST RECENT PERIOD (α)	2ND MOST RECENT PERIOD $\alpha(1-\alpha)$	3RD MOST RECENT PERIOD $\alpha(1-\alpha)^2$	4TH MOST RECENT PERIOD $\alpha(1-\alpha)^3$	5TH MOST RECENT PERIOD $\alpha(1-\alpha)^4$
$\alpha = .1$	.1	.09	.081	.073	.066
$\alpha = .5$	.5	.25	.125	.063	.031

Selecting the Smoothing Constant Exponential smoothing has been successfully applied in virtually every type of business. However, the appropriate value of the smoothing constant, α, can make the difference between an accurate forecast and an inaccurate forecast. High values of α are chosen when the underlying average is likely to change. Low values of α are used when the underlying average is fairly stable. In picking a value for the smoothing constant, the objective is to obtain the most accurate forecast.

STUDENT TIP
Forecasts tend to be more accurate as they become shorter. Therefore, forecast error also tends to drop with shorter forecasts.

Measuring Forecast Error

The overall accuracy of any forecasting model—moving average, exponential smoothing, or other—can be determined by comparing the forecasted values with the actual or observed values. If F_t denotes the forecast in period t, and A_t denotes the actual demand in period t, the *forecast error* (or deviation) is defined as:

$$\text{Forecast error} = \text{Actual demand} - \text{Forecast value}$$
$$= A_t - F_t$$

LO 4.4 *Compute* three measures of forecast accuracy

Several measures are used in practice to calculate the overall forecast error. These measures can be used to compare different forecasting models, as well as to monitor forecasts to ensure they are performing well. Three of the most popular measures are mean absolute deviation (MAD), mean squared error (MSE), and mean absolute percent error (MAPE). We now describe and give an example of each.

Mean Absolute Deviation The first measure of the overall forecast error for a model is the **mean absolute deviation (MAD)**. This value is computed by taking the sum of the absolute values of the individual forecast errors (deviations) and dividing by the number of periods of data (n):

Mean absolute deviation (MAD)
A measure of the overall forecast error for a model.

$$\text{MAD} = \frac{\sum |\text{Actual} - \text{Forecast}|}{n} \tag{4-5}$$

Example 4 applies MAD, as a measure of overall forecast error, by testing two values of α.

Most computerized forecasting software includes a feature that automatically finds the smoothing constant with the lowest forecast error. Some software modifies the α value if errors become larger than acceptable.

Example 4

DETERMINING THE MEAN ABSOLUTE DEVIATION (MAD)

During the past 8 quarters, the Port of Baltimore has unloaded large quantities of grain from ships. The port's operations manager wants to test the use of exponential smoothing to see how well the technique works in predicting tonnage unloaded. He guesses that the forecast of grain unloaded in the first quarter was 175 tons. Two values of α are to be examined: $\alpha = .10$ and $\alpha = .50$.

APPROACH ▶ Compare the actual data with the data we forecast (using each of the two α values) and then find the absolute deviation and MADs.

SOLUTION ▶ The following table shows the *detailed* calculations for $\alpha = .10$ only:

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST WITH $\alpha = .10$	FORECAST WITH $\alpha = .50$
1	180	175	175
2	168	175.50 = 175.00 + .10(180 − 175)	177.50
3	159	174.75 = 175.50 + .10(168 − 175.50)	172.75
4	175	173.18 = 174.75 + .10(159 − 174.75)	165.88
5	190	173.36 = 173.18 + .10(175 − 173.18)	170.44
6	205	175.02 = 173.36 + .10(190 − 173.36)	180.22
7	180	178.02 = 175.02 + .10(205 − 175.02)	192.61
8	182	178.22 = 178.02 + .10(180 − 178.02)	186.30
9	?	178.59 = 178.22 + .10(182 − 178.22)	184.15

To evaluate the accuracy of each smoothing constant, we can compute forecast errors in terms of absolute deviations and MADs:

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST WITH $\alpha = .10$	ABSOLUTE DEVIATION FOR $\alpha = .10$	FORECAST WITH $\alpha = .50$	ABSOLUTE DEVIATION FOR $\alpha = .50$
1	180	175	5.00	175	5.00
2	168	175.50	7.50	177.50	9.50
3	159	174.75	15.75	172.75	13.75
4	175	173.18	1.82	165.88	9.12
5	190	173.36	16.64	170.44	19.56
6	205	175.02	29.98	180.22	24.78
7	180	178.02	1.98	192.61	12.61
8	182	178.22	3.78	186.30	4.30
	Sum of absolute deviations:		82.45		98.62
	$\text{MAD} = \frac{\Sigma\lvert\text{Deviations}\rvert}{n}$		10.31		12.33

INSIGHT ▶ On the basis of this comparison of the two MADs, a smoothing constant of $\alpha = .10$ is preferred to $\alpha = .50$ because its MAD is smaller.

LEARNING EXERCISE ▶ If the smoothing constant is changed from $\alpha = .10$ to $\alpha = .20$, what is the new MAD? [Answer: 10.21.]

RELATED PROBLEMS ▶ 4.5b, 4.8c, 4.9c, 4.14, 4.23, 4.47b

Mean Squared Error The **mean squared error (MSE)** is a second way of measuring overall forecast error. MSE is the average of the squared differences between the forecasted and observed values. Its formula is:

Mean squared error (MSE)
The average of the squared differences between the forecasted and observed values.

$$\text{MSE} = \frac{\sum(\text{Forecast errors})^2}{n} \tag{4-6}$$

Example 5 finds the MSE for the Port of Baltimore problem introduced in Example 4.

The MSE tends to accentuate large deviations due to the squared term. For example, if the forecast error for period 1 is twice as large as the error for period 2, the squared error in period 1 is four times as large as that for period 2. Hence, using MSE as the measure of forecast error typically indicates that we prefer to have several smaller deviations rather than even one large deviation.

Mean Absolute Percent Error A problem with both the MAD and MSE is that their values depend on the magnitude of the item being forecast. If the forecast item is measured in thousands, the MAD and MSE values can be very large. To avoid

Example 5

DETERMINING THE MEAN SQUARED ERROR (MSE)

The operations manager for the Port of Baltimore now wants to compute MSE for $\alpha = .10$.

APPROACH ▶ Using the same forecast data for $\alpha = .10$ from Example 4, compute the MSE with Equation (4-6).

SOLUTION ▶

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST FOR $\alpha = .10$	$(\text{ERROR})^2$
1	180	175	$5^2 = 25$
2	168	175.50	$(-7.5)^2 = 56.25$
3	159	174.75	$(-15.75)^2 = 248.06$
4	175	173.18	$(1.82)^2 = 3.31$
5	190	173.36	$(16.64)^2 = 276.89$
6	205	175.02	$(29.98)^2 = 898.80$
7	180	178.02	$(1.98)^2 = 3.92$
8	182	178.22	$(3.78)^2 = 14.29$
			Sum of errors squared = 1,526.52

$$\text{MSE} = \frac{\sum(\text{Forecast errors})^2}{n} = 1{,}526.52/8 = 190.8$$

INSIGHT ▶ Is this MSE = 190.8 good or bad? It all depends on the MSEs for other forecasting approaches. A low MSE is better because we want to minimize MSE. MSE exaggerates errors because it squares them.

LEARNING EXERCISE ▶ Find the MSE for $\alpha = .50$. [Answer: MSE = 195.24. The result indicates that $\alpha = .10$ is a better choice because we seek a lower MSE. Coincidentally, this is the same conclusion we reached using MAD in Example 4.]

RELATED PROBLEMS ▶ 4.8d, 4.11c, 4.14, 4.15c, 4.16c, 4.20

Mean absolute percent error (MAPE)
The average of the absolute differences between the forecast and actual values, expressed as a percent of actual values.

this problem, we can use the **mean absolute percent error (MAPE)**. This is computed as the average of the absolute difference between the forecasted and actual values, expressed as a percentage of the actual values. That is, if we have forecasted and actual values for n periods, the MAPE is calculated as:

$$\text{MAPE} = \frac{\sum_{i=1}^{n} 100|\text{Actual}_i - \text{Forecast}_i|/\text{Actual}_i}{n} \quad (4\text{-}7)$$

Example 6 illustrates the calculations using the data from Examples 4 and 5.

The MAPE is perhaps the easiest measure to interpret. For example, a result that the MAPE is 6% is a clear statement that is not dependent on issues such as the magnitude of the input data.

Table 4.1 summarizes how MAD, MSE, and MAPE differ.

Exponential Smoothing with Trend Adjustment

Simple exponential smoothing, the technique we just illustrated in Examples 3 to 6, is like any other moving-average technique: It fails to respond to trends. Other forecasting techniques that can deal with trends are certainly available. However, because exponential smoothing is such a popular modeling approach in business, let us look at it in more detail.

Example 6

DETERMINING THE MEAN ABSOLUTE PERCENT ERROR (MAPE)

The Port of Baltimore wants to now calculate the MAPE when $\alpha = .10$.

APPROACH ▶ Equation (4-7) is applied to the forecast data computed in Example 4.

SOLUTION ▶

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST FOR $\alpha = .10$	ABSOLUTE PERCENT ERROR 100 (|ERROR|/ACTUAL)
1	180	175.00	100(5/180) = 2.78%
2	168	175.50	100(7.5/168) = 4.46%
3	159	174.75	100(15.75/159) = 9.90%
4	175	173.18	100(1.82/175) = 1.05%
5	190	173.36	100(16.64/190) = 8.76%
6	205	175.02	100(29.98/205) = 14.62%
7	180	178.02	100(1.98/180) = 1.10%
8	182	178.22	100(3.78/182) = 2.08%
			Sum of % errors = 44.75%

$$\text{MAPE} = \frac{\Sigma \text{ absolute percent error}}{n} = \frac{44.75\%}{8} = 5.59\%$$

INSIGHT ▶ MAPE expresses the error as a percent of the actual values, undistorted by a single large value.

LEARNING EXERCISE ▶ What is MAPE when α is .50? [Answer: MAPE = 6.75%. As was the case with MAD and MSE, the $\alpha = .1$ was preferable for this series of data.]

RELATED PROBLEMS ▶ 4.8e, 4.29c

TABLE 4.1 Comparison of Measures of Forecast Error

MEASURE	MEANING	EQUATION	APPLICATION TO CHAPTER EXAMPLE
Mean absolute deviation (MAD)	How much the forecast missed the target	$\text{MAD} = \frac{\sum \lvert \text{Actual} - \text{Forecast} \rvert}{n}$ (4-5)	For $\alpha = .10$ in Example 4, the forecast for grain unloaded was off by an average of 10.31 tons.
Mean squared error (MSE)	The square of how much the forecast missed the target	$\text{MSE} = \frac{\sum (\text{Forecast errors})^2}{n}$ (4-6)	For $\alpha = .10$ in Example 5, the square of the forecast error was 190.8. This number does not have a physical meaning but is useful when compared to the MSE of another forecast.
Mean absolute percent error (MAPE)	The average percent error	$\text{MAPE} = \frac{\sum_{i=1}^{n} 100 \lvert \text{Actual}_i - \text{Forecast}_i \rvert / \text{Actual}_i}{n}$ (4-7)	For $\alpha = .10$ in Example 6, the forecast is off by 5.59% on average. As in Examples 4 and 5, some forecasts were too high, and some were low.

Here is why exponential smoothing must be modified when a trend is present. Assume that demand for our product or service has been increasing by 100 units per month and that we have been forecasting with $\alpha = 0.4$ in our exponential smoothing model. The following table shows a severe lag in the second, third, fourth, and fifth months, even when our initial estimate for month 1 is perfect:

MONTH	ACTUAL DEMAND	FORECAST (F_t) FOR MONTHS 1–5
1	100	$F_1 = 100$ (given)
2	200	$F_2 = F_1 + \alpha(A_1 - F_1) = 100 + .4(100 - 100) = 100$
3	300	$F_3 = F_2 + \alpha(A_2 - F_2) = 100 + .4(200 - 100) = 140$
4	400	$F_4 = F_3 + \alpha(A_3 - F_3) = 140 + .4(300 - 140) = 204$
5	500	$F_5 = F_4 + \alpha(A_4 - F_4) = 204 + .4(400 - 204) = 282$

To improve our forecast, let us illustrate a more complex exponential smoothing model, one that adjusts for trend. The idea is to compute an exponentially smoothed average of the data and then adjust for positive or negative lag in trend. The new formula is:

$$\text{Forecast including trend } (FIT_t) = \text{Exponentially smoothed forecast average } (F_t) + \text{Exponentially smoothed trend } (T_t) \quad (4\text{-}8)$$

With trend-adjusted exponential smoothing, estimates for both the average and the trend are smoothed. This procedure requires two smoothing constants: α for the average and β for the trend. We then compute the average and trend each period:

$$F_t = \alpha(\text{Actual demand last period}) + (1 - \alpha)(\text{Forecast last period} + \text{Trend estimate last period})$$

or:

$$F_t = \alpha(A_{t-1}) + (1 - \alpha)(F_{t-1} + T_{t-1}) \quad (4\text{-}9)$$

$$T_t = \beta(\text{Forecast this period} - \text{Forecast last period}) + (1 - \beta)(\text{Trend estimate last period})$$