Final Term Examination Spring Semester- 2021

CONTROL SYSTEM

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Program: BSCS

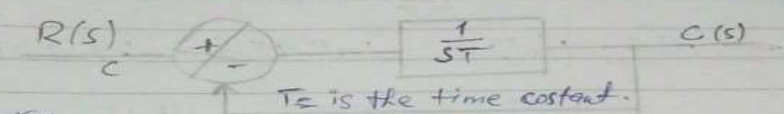
Q1. Derive the time response equations of transfer function C(t) for

1. Unit step function.
2. Unit impulse function

Answer:

QUESTION NO:-1

A — UNIT STEP FUNCTION:- Date: 31/Aug/2021



Where as,
C(S) = laplace Transform of the output signal c(t) R(s) = is the laplace transform input signal.

$$T(s) = \frac{G(s)}{R(s)} = \frac{1}{1+ST}$$

$$\frac{G(s)}{R(s)} = \frac{1}{1+ST}$$

$$C(s) = R(s)\frac{1}{1+ST} = \frac{1}{S} \times \frac{1}{1+ST} = \frac{1-ST}{S} \quad \frac{1/T^2}{1/T+s}$$

$$C(s) = \frac{1}{S} - \frac{1}{(S+1/T)}$$

$$C(t) = [C^{-1}\ 1/s - 1/s + 1/T]$$

$$= 1-e - 1/t$$

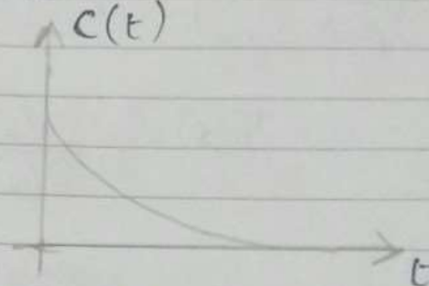
$$\# \ t == 0.$$

Date:

B- THE UNIT IMPULSE RESPONSE :-

$$T(s) = \frac{C(s)}{R(s)} = \frac{1/sT}{1 + 1/sT = 1}$$

$$= \frac{1}{1+sT}$$



$$r(t) = \delta(t)$$

$$R(s) = \alpha T[r(t)] = LT[\delta(t)] = 1$$

$$\frac{C(s)}{R(s)} = \frac{1}{1+sT}$$

$$C(s) = R(s)\frac{1}{1+sT} = \frac{1}{1+sT}$$

$$C(s) = \frac{1/T}{(1/T + s)}$$

$$C(t) = L\left[C(s) = L^{-1}\left[(1/T\left(\frac{1}{s+1/T}\right)\right]\right.$$

$$C(t) = 1/T\, e^{-at}$$

$$\{t \geq 0\}.$$

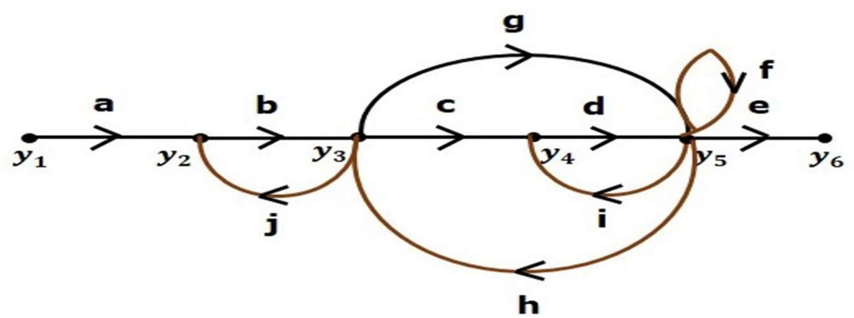
Q2. Consider the following signal flow graph.

1. Find the nth farward paths and
2. Calculate Transfer function using Meson's gain formula.

Answer:

Calculation of Transfer Function using Mason's Gain Formula

Let us consider the same signal flow graph for finding transfer function.



- Number of forward paths, N = 2.
- First forward path is - $y_1 \rightarrow y_2 \rightarrow y_3 \rightarrow y_4 \rightarrow y_5 \rightarrow y_6$.
- First forward path gain, $p_1 = abcde$.
- Second forward path is - $y_1 \rightarrow y_2 \rightarrow y_3 \rightarrow y_5 \rightarrow y_6$.
- Number of individual loops, L = 5.
- Loops are - $y_2 \rightarrow y_3 \rightarrow y_2$, $y_3 \rightarrow y_5 \rightarrow y_3$, $y_3 \rightarrow y_4 \rightarrow y_5 \rightarrow y_3$, $y_4 \rightarrow y_5 \rightarrow y_4$ and $y_5 \rightarrow y_5$.
- Loop gains are - $l_1 = bj$, $l_2 = gh$, $l_3 = cdh$, $l_4 = di$ and $l_5 = f$.
- Number of two non-touching loops = 2.
- First non-touching loops pair is - $y_2 \rightarrow y_3 \rightarrow y_2$, $y_4 \rightarrow y_5 \rightarrow y_4$.
- Gain product of first non-touching loops pair, $l_1 l_4 = bjdi$
- Second non-touching loops pair is - $y_2 \rightarrow y_3 \rightarrow y_2$, $y_5 \rightarrow y_5$.
- Gain product of second non-touching loops pair is - $l_1 l_5 = bjf$

Higher number of (more than two) non-touching loops are not present in this signal flow graph.

We know,

$$\Delta = 1 - (sum\ of\ all\ individual\ loop\ gains)$$

$$+(sum\ of\ gain\ products\ of\ all\ possible\ two\ nontouching\ loops)$$

$$-(sum\ of\ gain\ products\ of\ all\ possible\ three\ nontouching\ loops) + \ldots$$

Substitute the values in the above equation,

$$\Delta = 1 - (bj + gh + cdh + di + f) + (bjdi + bjf) - (0)$$

$$\Rightarrow \Delta = 1 - (bj + gh + cdh + di + f) + bjdi + bjf$$

There is no loop which is non-touching to the first forward path.

So, $\Delta_1 = 1$.

Similarly, $\Delta_2 = 1$. Since, no loop which is non-touching to the second forward path.

Substitute, N = 2 in Mason's gain formula

$$T = \frac{C(s)}{R(s)} = \frac{\Sigma_{i=1}^{2} P_i \Delta_i}{\Delta}$$

$$T = \frac{C(s)}{R(s)} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

Substitute all the necessary values in the above equation.

$$T = \frac{C(s)}{R(s)} = \frac{(abcde)1 + (abge)1}{1 - (bj + gh + cdh + di + f) + bjdi + bjf}$$

$$\Rightarrow T = \frac{C(s)}{R(s)} = \frac{(abcde) + (abge)}{1 - (bj + gh + cdh + di + f) + bjdi + bjf}$$

Therefore, the transfer function is -

$$T = \frac{C(s)}{R(s)} = \frac{(abcde) + (abge)}{1 - (bj + gh + cdh + di + f) + bjdi + bjf}$$

Q3. Elaborate the responses of control system in time domain. Also the transfer function.

Answer:

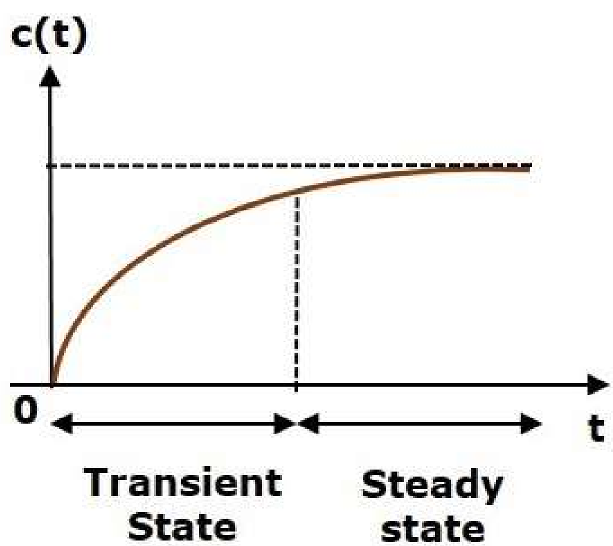
Control System in Time Domain

The responses of control system in time domain, if the output of control system for an input varies with respect to time, then it is called the time response of the control system.

The time response consists of two parts.

1. Transient response
2. Steady state response

The response of control system in time domain is shown in the following figure.



Here, both the transient and the steady states are indicated in the figure. The responses corresponding to these states are known as transient and steady state responses.

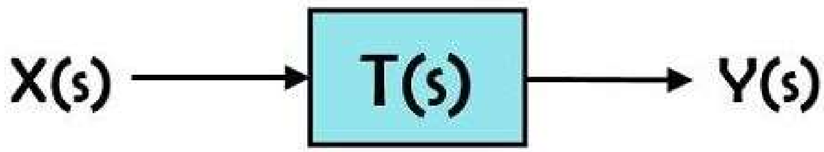
Mathematically, we can write the time response c(t) as

$$c(t) = c_{tr}(t) + c_{ss}(t)$$

Transfer Function:

The transfer function of a control system is the ratio of Laplace transform of output to that of the input while taking the initial conditions, as 0. Basically it provides a relationship between input and output of the system.

For a control system, T(s) generally represents the transfer function. In the figure given below X(s) and Y(s) represents input and output respectively.



The transfer of the system is given as:

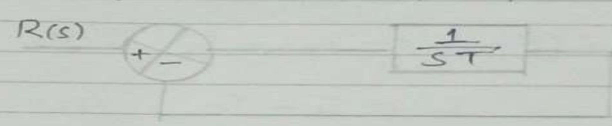
$$T(s) = \frac{Laplace\ transform\ of\ output}{Laplace\ transform\ of\ input} = \frac{Y(s)}{X(s)}$$

Q4. Prove that the Time response of first order unit ramp signal is Ƭ.

Answer:

QUESTION NO:- 04

TIME RESPONSE OF FIRST ORDER UNIT RAMP sig



Consider the unit ramp as an input to the first order system.

So, $r(t) = tu(t)$

Apply Laplace transform on both sides:

$R(s) = \frac{1}{S^2}$

Equation

$C(s) = \left(\frac{1}{ST+1}\right) R(s)$

$\frac{C(s)}{R(s)} = \frac{1}{1+ST}$

Applying substitute formula of PFR.

$C(S) = R(S) \frac{1}{1+ST} = \frac{1}{S^2} \frac{1}{1+ST} = \frac{1-ST}{S^2} + \frac{T^2}{1+ST}$ $T\left(\frac{1}{T} + s\right.$

$$C(s) = \frac{1}{s^2} - T\frac{1}{s} + T\frac{1}{(s+1/T)}$$

I.L.T on both sides

$$c(t) = t - T \cdot 1 + Te^{-1/st}$$

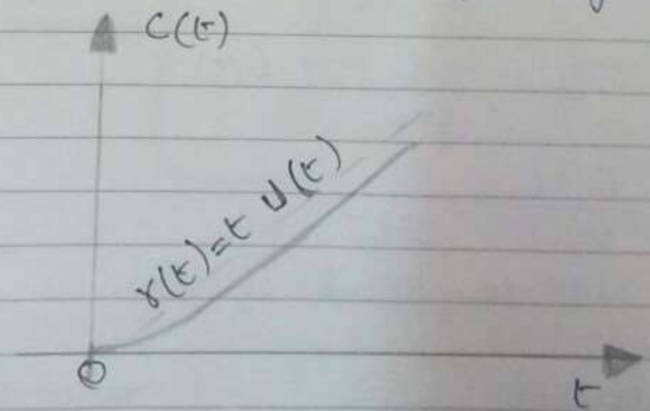
$$= t - T + Te^{-1/Tt}$$

Study state error. (ess).

$$= (t\,e(t) = (t(T - Te^{-1/\tau t})$$

$$= t - \alpha = t - \alpha \Rightarrow T$$

The unit ramp response, c(t) follows the unit ramp input signal for all positive values of t. But there is a derivation of T units from the input signal.



THANK YOU! :)